Exo 1

(1) $A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 3 & 4 & 2 & 3 \\ -2 & 0 & 4 & -8 \\ 1 & 0 & -2 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & -3 \\ 0 & 2 & 4 & -4 \\ 0 & -1 & -2 & -2 \end{pmatrix}$

$\rightsquigarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -5 \end{pmatrix}$ having determinant $= 1 \cdot 1 \cdot 0 \cdot (-5) = 0$.

So $\det A = 0$ and A is singular.

(2)

$$A \rightsquigarrow \begin{array}{c} \begin{array}{cccc} \downarrow & \downarrow & & \downarrow \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

basic variables

, so rank $A = 3$.

(3) So a basis for column space of A is

$$\begin{pmatrix} 1 \\ 3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -8 \\ 0 \end{pmatrix}$$

and for row space of A

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}.$$

And kernel of A :

$$\begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

Exo 2

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 4 & 3 & 10 \\ 1 & 2 & 5 \\ 6 & 1 & 8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 2 & -1 & 0 \\ 0 & 5 & 10 \\ 0 & 5/2 & 5 \\ 0 & 4 & 8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 2 & -1 & 0 \\ 0 & 5 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and a basis consists of $\begin{pmatrix} 2 \\ 4 \\ 1 \\ 6 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 2 \\ 1 \end{pmatrix}$.

Exo 3

$$\det \begin{pmatrix} 4 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 3 & 1 \end{pmatrix} = 4\left(2\cdot 1 + 3\cdot 2\right) - 1\left(2\cdot 1 + 1\cdot 2\right) - 1\left(2\cdot 3 - 1\cdot 2\right)$$

$$= 4\cdot 8 - 4 - 4 = 24 \neq 0$$

This means that the matrix $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 3 & 1 \end{pmatrix}$

is invertible and so the three polynomials p_1, p_2, p_3 are linearly indep.

Exo 4

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1/2 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1/2 & 2 \\ 0 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

So $L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 2 & & 0 \\ & 1/2 & \\ 0 & & -2 \end{pmatrix}$.