

**TITLE AND ABSTRACTS FOR THE 2012 ATKIN  
WORKSHOP**

1. Jennifer Balakrishnan, *p-adic heights on elliptic curves.*

*Abstract:* In 2006, the work of Mazur-Stein-Tate gave an effective algorithm to compute  $p$ -adic heights on elliptic curves over  $\mathbb{Q}$ . We begin with a brief overview of their algorithm and discuss a generalization to number fields. This is joint work with Mirela Ciperiani and William Stein.

2. Jonathan Bober, *Searching for [equations of] elliptic curves over  $\mathbb{Q}(\sqrt{5})$ .*

*Abstract:* In making complete tables of elliptic curves over  $\mathbb{Q}(\sqrt{5})$ , we need a way to make a list of equations of curves from a list of Hilbert modular forms. There seems to be no known efficient algorithm for solving this problem in all cases, but there are many strategies which will work well sometimes. I will describe the variety of techniques that were used to make a list of the first (ordered by conductor norm) 1414 isogeny classes of curves over  $\mathbb{Q}(\sqrt{5})$ .

This is joint work with Alyson Deines, Aariah Klages-Mundt, Benjamin LeVeque, R. Andrew Ohana, Ashwath Rabindranath, Paul Sharaba, and William Stein.

3. John Cremona, *Elusive isogenies and unusual modular curves.*

*Abstract:* In a recent paper, Sutherland asked the question, to what extent is the existence of a rational  $\ell$ -isogeny for a given elliptic curve  $E$  defined over a number field  $K$  a “local” phenomenon, in the sense that  $E$  possesses such an isogeny if and only if the reduction of  $E$  modulo  $p$  does for almost all primes  $p$  of  $K$ . Sutherland establishes a criterion for this, which over  $\mathbb{Q}$  is satisfied for all  $\ell$  and all elliptic curves, with the single exception of  $\ell = 7$  and curves  $E$  with  $j$ -invariant  $2268945/128$ , but which relies on the ground field not containing the quadratic subfield of the  $\ell$ th cyclotomic field. We give an infinite set

of counterexamples for  $\ell = 5$  over  $\mathbb{Q}(\sqrt{5})$ , parametrised by a genus 0 modular curve of level 5, which is a moduli space for elliptic curves whose projective mod 5 representation has image isomorphic to  $V_4$ . We will also report on progress towards finding similar examples with  $\ell = 13$  associated to a modular curve of genus 3 and level 13, via the closely related problem (which may be of independent interest) of finding explicit models for modular curves parametrising elliptic curves whose projective mod  $\ell$  representation has image isomorphic to  $A_4$  or  $A_5$ . This is joint work with Barinder Banwait.

4. Lassina Dembele, *Galois representations and equations of hyperelliptic curves.*

*Abstract:* This is a progress report. We will present an algorithm which, given a Hilbert newform  $f$  of parallel weight 2 such that the field of Fourier coefficients  $K_f$  is at most quadratic and the associated mod 2 residual Galois representation is surjective, finds the corresponding hyperelliptic curve whose Jacobian has RM by  $K_f$ . This construction assumes the Eichler-Shimura conjecture.

5. Noam Elkies, *Remarks on isogenies over  $\mathbb{Q}(\sqrt{5})$  and other number fields.*

*Abstract:* On the occasion of the creation of a table of modular elliptic curves over  $\mathbb{Q}(\sqrt{5})$ , we review the “Remarks on isogenies” that accompanied the “Antwerp” tables (LNM 476), and outline some of the new phenomena and open questions that arise in attempting to give a similar overview of isogenies defined over  $\mathbb{Q}(\sqrt{5})$  or other number fields. In particular, we account for some new isogeny degrees and graphs not seen over  $\mathbb{Q}$ , and explain why the problem of proving completeness of the list over  $\mathbb{Q}(\sqrt{5})$  is difficult but not hopeless.

6. Edray Goins, *There Exist an Elliptic Curve  $E/\mathbb{Q}$  with Mordell-Weil Group  $Z_2 \times Z_8 \times \mathbb{Z}^4$ ?*

*Abstract:* An elliptic curve  $E$  defined over the rational numbers  $\mathbb{Q}$  is an arithmetic-algebraic object: It is simultaneously a nonsingular projective curve with an affine equation  $Y^2 = X^3 + AX + B$ , which

allows one to perform arithmetic on its points; and a finitely generated abelian group  $E(\mathbb{Q}) \simeq E(\mathbb{Q})_{\text{tors}} \times \mathbb{Z}^r$ , which allows one to apply results from abstract algebra. The abstract nature of its rank  $r$  can be made explicit by searching for rational points  $(X, Y)$ .

The largest possible subgroup of an elliptic curve  $E$  is  $E(\mathbb{Q})_{\text{tors}} \simeq Z_2 \times Z_8$ , and, curiously, these curves seem to have the least known information about the rank  $r$ . To date, there are twenty-seven known examples of elliptic curves over  $\mathbb{Q}$  having Mordell-Weil group  $E(\mathbb{Q}) \simeq Z_2 \times Z_8 \times \mathbb{Z}^3$ , yet no larger rank has been found.

In this talk, we give some history on the problem of determining properties of  $r$ , explain its importance by discussing the conjecture of Birch and Swinnerton-Dyer, and analyze various approaches to finding curves of large rank.

7. Matthew Greenberg, *Definite quaternion algebras and triple product  $p$ -adic  $L$ -functions*.

*Abstract:* Formulas of Gross–Kudla, Boecherer–Schulze–Pillot, and Ichino express central values of triple product  $L$ -functions in terms of trilinear forms evaluated on specific test vectors. I will discuss joint work with Marco Seveso in which we show that, in the “definite case,” these trilinear forms can be  $p$ -adically interpolated, giving rise to triple product  $p$ -adic  $L$ -functions.

8. Richard Pinch, *Elliptic curves with good reduction away from 2*.

*Abstract:* We show how to determine the complete list of elliptic curves over  $\mathbb{Q}(\sqrt{5})$  with good reduction away from the prime 2. The computation makes use of Baker’s method and techniques for verifying that a list of solutions to a Diophantine equation is complete.

9. Kenneth Ribet, *TBA*.

10. William Stein, *A Database of Elliptic Curves over  $\mathbb{Q}(\sqrt{5})$ —First Report*.

*Abstract:* I will describe a tabulation of (conjecturally) modular elliptic

curves over the field  $\mathbf{Q}(\sqrt{5})$  up to the first curve of rank 2. Using an efficient implementation of an algorithm of Lassina Dembele, we computed tables of Hilbert modular forms of weight  $(2, 2)$  over  $\mathbf{Q}(\sqrt{5})$ , and via a variety of methods we constructed corresponding elliptic curves, including (again, conjecturally) all elliptic curves over  $\mathbf{Q}(\sqrt{5})$  that have conductor with norm less than or equal to 1831.

11. Nike Vatsal, *Some number theory associated to modular forms on  $SL(2)$* .

*Abstract:* We'll discuss some more or less well-known results on automorphic forms on  $SL(2)$ , and give some arithmetic consequences (which may be less well-known). The main idea is to explain how some familiar questions from number theory maybe very naturally formulated in terms of automorphic forms on  $SL(2)$ , rather than the usual group  $GL(2)$

12. John Voight, *Computing power series expansions of modular forms*.

*Abstract:* We exhibit an method to numerically compute power series expansions of modular forms on a cocompact Fuchsian group using the explicit computation a fundamental domain and linear algebra.