Math 215 —- Exam I

Spring 2009

All problems are weighted equally. Partial credit will be given only if your answer makes sense.

1. Write the contrapositive of the following statement, and prove it:
For any integer n, if 3 does not divide $n^2 + n$ then 3 does not divide n.

2. Prove by induction that

$$(1 - \frac{1}{2^2})(1 - \frac{1}{3^2}) \dots (1 - \frac{1}{n^2}) = \frac{n+1}{2n}$$

(For the base case, take $n = 2$.)

3. Prove the following statement, after filling in the values for a and b on the right hand side. Show all your steps.

$$\{x \in \mathbf{R} | x^2 - 7x + 12 > 0\} = \{x \in \mathbf{R} | x > a \} \cup \{x \in \mathbf{R} | x < b \}$$

4. Prove or disprove the following statements:
(i) $\forall x \in \mathbf{R}^+, \exists y \in \mathbf{R},\ x = y^2$
(ii) $\exists x \in \mathbf{R}^+, \forall y \in \mathbf{R},\ x = y^2$
(iii) $\forall x \in \mathbf{R}, \exists y \in \mathbf{R}^+,\ x = y^2$
(iv) $\forall r \in \mathbf{R}^+, \exists x, y \in \mathbf{R}^+, (x^2 + y^2 \leq r^2)$ [Draw a picture in $\mathbf{R^2}$.]

5.(a) Find the composites $f.g$ and $g.f$ of the following functions f and g. Find the elements $x \in \mathbf{R}$ such that $(f.g)(x) = (g.f)(x)$.

$f : \mathbf{R} \to \mathbf{R},\ f(x) = x^3 - 6$, and $g : \mathbf{R} \to \mathbf{R},\ g(x) = 2x$.

(b) For the following function f, find whether f is injective and whether it is surjective. If f is not surjective find $\mathrm{Im} f$.
$f : \mathbf{R} \to \mathbf{R},\ f(x) = |x| + 1$.