

Math 215 —-Sample Exam I

Spring 2009

All problems are weighted equally. Partial credit will be given only if your answer makes sense.

1. (i) Prove:

(a) If $a|b$ and $a|c$ then $a|(b+c)$.

(b) If $a|b$ or $a|c$ then $a|bc$.

(ii) Write the converse of each of the above statements (a) and (b) and either prove or disprove the converse.

2. Prove by induction that

$$\sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. If A, B, C, D are sets show that

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

4. Prove or disprove the following statements:

(i) $\forall x \in \mathbf{R}^+, \exists y \in \mathbf{R}^+, xy = 1$

(ii) $\exists x \in \mathbf{R}^+, \forall y \in \mathbf{R}^+, xy = 1$

(iii) $\forall x \in \mathbf{R}, \forall y \in \mathbf{R}, (x+y \geq 0 \text{ or } x+y \leq 0)$

(iv) $(\forall x \in \mathbf{R}, \forall y \in \mathbf{R}, x+y \geq 0) \text{ or } (\forall x \in \mathbf{R}, \forall y \in \mathbf{R}, x+y \leq 0)$

5. (i) For the following functions f , find whether f is injective and whether it is surjective. If f is not surjective find $\text{Im}f$.

(a) $f : \mathbf{R} \rightarrow \mathbf{R}, f(x) = e^x + 2$.

(b) $f : \mathbf{R}^2 \rightarrow \mathbf{R}, f((x, y)) = x + y$.

(ii) Find the composites $f.g$ and $g.f$ of the following functions f and g . Find the elements $x \in \mathbf{R}$ such that $(f.g)(x) = (g.f)(x)$.

$f : \mathbf{R} \rightarrow \mathbf{R}, f(x) = \sqrt{1+x}$, and $g : \mathbf{R} \rightarrow \mathbf{R}, g(x) = x^2$.