

## Math 215 —Sample Exam 2

Spring 2009

All problems are weighted equally. Partial credit will be given only if your answer makes sense. You must give reasons for your answers.

- (a) If  $X, Y$  are sets of  $m$  and  $n$  elements respectively, state how many (i) functions (ii) injective functions there are from  $X$  to  $Y$ .

(b) Find the number of permutations (bijective functions)  $X \rightarrow X$  where  $X = \{1, 2, 3, 4\}$  such that  $f(i) \neq i$  for any  $i$ . [Hint: Find those functions for which  $f(i) = i$  for some  $i$ , and subtract from the total number. See the problem 17 on p.185 of the text.]
- (a) State the Binomial Theorem.

(b) Show that  $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$ .
- (a) Define the greatest common divisor  $d$  of two non-zero integers  $a$  and  $b$ .

(b) Let  $a, b$  be non-zero even integers such that  $\gcd(a, b) = 2$ . Show that  $\gcd(a/2, b/2) = 1$ .
- A committee of 7 is to be formed from a group of 10 people.

(a) At least one of two individuals A and B is to be on the committee. Let X be the set of those committees of 7 which include A and let Y be the set of those committees of 7 which include B. Find the number of committees which include either A or B (perhaps both). Express the set of these committees in terms of X and Y.

(b) Suppose any committee member can fill more than one slot on the committee. (For example, assume 7 areas are represented and all 10 people are experts in the 7 areas.) How many committees can be formed?
- (a) Using congruences, show that an integer of the form  $521 \cdot 12^k + 1$ ,  $k \geq 1$ ,  $k$  odd is divisible by 13.

(b) Find the remainders mod 7, 11 and 13 of 1111118111111. State the theorem you are using here.