Math 310 Exam 2

Name: **November 9, 2001**

Answer all **5** questions completely. You must **show all work** to receive credit. If you use a calculator, please write *calculator calculation* where it was used. Try to check your solutions.

1. Given the transformation $L : R^2 \longrightarrow R^2$ defined by

$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 + 2x_2 \\ 5x_1 \end{pmatrix}.$$

   (a) Verify that L is a linear transformation.

   (b) Find a basis for the $ker(L)$. What is its dimension?

   (c) Find a basis for the range of L.

2. Given the transformation $L : R^2 \longrightarrow R^2$ defined by

$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 + 2x_2 \\ 5x_1 \end{pmatrix}.$$

   (a) Find a matrix representation, A, of L using the standard basis in R^2.

   (b) Find a matrix representation, B, of L using the basis $E = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$, i.e. $B[\mathbf{u}]_E = [\mathbf{w}]_E$.

3. (a) Let S be the span of $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$ in R^3. Find a basis for $S^\perp$ and give the dimension of $S^\perp$.

   (b) State the Fredholm alternative theorem for the problem $A\mathbf{x} = \mathbf{b}$.

   (c) Solve $A^T\mathbf{x} = 0$ and use the theorem in (b) (*required for full credit*) to find the value of a such that $A\mathbf{x} = \mathbf{b}$ has at least one solution where $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ a^2 - 2 \end{pmatrix}$

4. Consider $A\mathbf{x} = \mathbf{b}$ given by $\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$

   (a) Is $\mathbf{b}$ in the range $R(A)$?

   (b) Find the least squares solution $\hat{\mathbf{x}}$ of $A\mathbf{x} = \mathbf{b}$.

   (c) Find the element $\mathbf{p}$ in $R(A)$ that is closest to $\mathbf{b}$.

5. (a) Given subspace S with basis $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}$ in R^3. Use Gram-Schmidt process to find an orthonormal basis for S.

   (b) Find the projection of the vector $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ onto the subspace S in (a).