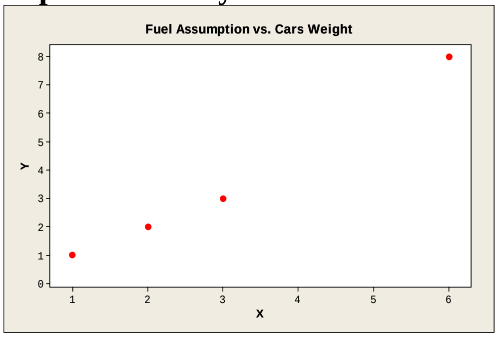
STAT 381 - 13605 Quiz 2 2006-9-13

Solution

1. Consider the weight x (in 1000 pounds) and the fule consumption y (gallons per 100 miles) of 4 cars in the following table.

Car weight x	3	1	2	6
Fuel Consumption y	3	1	2	8

(1) (3pt) Draw the scatter plot of x and y.



(2) Compute the sample correlation coefficient.

	x	y	$(x_i - \vec{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
Car 1	3	3	0	0	.25
Car 2	1	1	5	4	6.25
Car 3	2	2	1.5	1	2.25
Car 4	6	8	13.5	9	20.25
Total Sum	12	14	20	14	29

$$\bar{x} = \frac{1}{n}\sum x_i = \frac{12}{4} = 3, \quad s_x = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2} = \sqrt{\frac{14}{3}} = 2.160$$

$$\bar{y} = \frac{1}{n}\sum y_i = \frac{14}{4} = 3.5, \quad s_y = \sqrt{\frac{1}{n-1}\sum (y_i - \bar{y})^2} = \sqrt{\frac{29}{3}} = 3.109$$

Sample correlation coefficient $r = \frac{\sum (x_i - \vec{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \vec{x})^2 \sum (y_i - \bar{y})^2}} = \frac{20}{\sqrt{14 * 29}} = .993$

(3) By the method of least squares, fit $y = \alpha + \beta x$ to the data given above.

Estimate of slope $\hat{\beta} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{20}{14} = 1.429$,

or $\hat{\beta} = r \cdot \frac{s_y}{s_x} = 0.993 \cdot \frac{3.109}{2.160} = 1.429$

Estimate of intercept

$$\hat{\alpha} = \bar{y} - \hat{\beta} \cdot \bar{x} = 3.5 - (1.429 * 3) = -0.787$$

Hence the best linear fitted line is $y = -0.787 + 1.429 \cdot x$