STAT 381 - 13605 Quiz 5 2006-10-18

Solution

1. **[6pt] Suppose the probability of producing a defective item is 0.05.**

a). What is the probability that exactly two is defective out of 10 items that are selected independently?

Let X be the number of defective ones in the 10 items, then $X \sim B(10, 0.05)$

{X=2} = { exactly two is defective out of 10 items }

Use the Binomial Table : $P(X = 2) = F(2) - F(1) = .9885 - .9139 = .0746$

where F(x) is the cumulative distribution function of the random variable X.

Or directly $P(X = 2) = \binom{10}{2} 0.05^2 (1 - 0.05)^8 = .0746$

b). How many items do we have to produce so that the probability of producing at least one defective item is greater than 90%?

P(X ≥ 1) ≥ .90, then 1- P(X=0) ≥ .90

P(X=0) = 0.95^n, then $1 - 0.95^n \geq 0.90$ i.e. $0.95^n \leq 0.10$

$n \geq \log(0.10)/\log(0.95)$, so n ⩾44.89 ↗ n=45

2. **[4pt] The daily number of plant shutdown follows a Poisson distribution with mean 2.**

a) What is the probability that there is at least one shutdown in a day?

Let Y be the daily number of plant shutdown, then $Y \sim P(2)$

{Y ≥ 1} = { at least one shutdown in a day }

Use the Poisson Table : $P(Y \geq 1) = 1 - F(0) = 1 - .135 = .865$

Or $P(Y \geq 1) = 1 - P(X = 0) = 1 - \frac{2^0 \cdot e^{-2}}{0!} = 1 - e^{-2} = 1 - 0.1353 = 0.8647$

b) Assume that the company losses $ 10000 on each shutdown. Calculate the expected daily loss.

Let Z be the daily loss of the company, Z = 10000*Y

Since $Y \sim P(2)$, then E(Y) = 2.

Hence the expected daily loss E(Z) = 10000*E(Y) =20000.