STAT 381 - 13605 **Quiz 9** **2006-11-22**

Solution

1. **A tire manufacturer tests n=144 tires and finds their mean life to be $\bar{x}$ =40,200 miles. The population standard deviation is $\sigma = 1250$ miles. Find a 99% confidence interval for the average life of these tires.**

   1-α = 99%, α = 0.01, α /2= 0.005, then $z_{0.005} = 2.576$

   $$\bar{x} \pm z_{0.005} \frac{\sigma}{\sqrt{n}} = 40200 \pm \left(2.576 \cdot \frac{1250}{\sqrt{144}} \right) = 40200 \pm 268.3 = (39931.7, 40468.3)$$

2. **Let Y be a binomial random variable b(n=25, p=0.2), use the normal distribution to approximate the probability $P(4 \le Y \le 8)$.**

   Mean μ_Y = np =5, Standard deviation $\sigma_Y = \sqrt{np(1-p)} = \sqrt{25 \cdot 0.2 \cdot 0.8} = 2$

   $$\begin{aligned} &P(4 \le Y \le 8) = P(4 - 0.5 \le Y \le 8 + 0.5) \\ &= P(3.5 \le Y \le 8.5) = P\left(\frac{3.5 - \mu_Y}{\sigma_Y} \le \frac{Y - \mu_Y}{\sigma_Y} \le \frac{8.5 - \mu_Y}{\sigma_Y} \right) \\ &\cong P\left(\frac{3.5 - 5}{2} \le Z \le \frac{8.5 - 5}{2} \right) = P(-0.75 \le Z \le 1.75) \\ &= \Phi(1.75) - [\Phi(-0.75)] = 0.9599 - (1 - 0.7734) = 0.7333 \end{aligned}$$

3. **A telephone company wants to estimate the mean number of minutes people in a city spend talking long distance with 95% confidence. From past records, an estimate of the standard deviation is $\sigma = 12$ minutes. What is the minimum sample size required if the desired margin of error is 5 minutes?**

   Confidence 1-α = 95%, α = 0.05, α /2= 0.025, then $z_{0.025} = 1.96$
   Margin of error B=5

   $$n \ge \frac{z_{\alpha/2}^2 \cdot \sigma^2}{B^2} = \frac{1.96^2 \cdot 12^2}{5^2} = 22.13 \quad \nearrow 23.$$