Spring 2003 Name ________________

MCS 261:01 Hour Exam 2

1. Write you solutions in your exam book.
2. Turn in this sheet along with your exam book.
3. Show and explain all your work. An unjustified answer may not receive credits.

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1. Determine, with a short justification, whether each of the following sets is finite, countably infinite, or uncountable (20 points):
   (a) $\{x \in \mathbf{Q} : 0 < x < 1\}$
   (b) $\{(a, b) \in \mathbf{R} \times \mathbf{R} : a + b = 1\}$
   (c) $\{\frac{m}{n} : m, n \in \mathbf{N}, m < 5, n > 5\}$
2. Use mathematical induction to show that $(1 + \frac{1}{2})^n \geq 1 + \frac{n}{2}$. (20 points)
3. Let a_n be defined recursively by $a_1 = 1, a_2 = 0$ and for $n > 2$, $a_n = 4a_{n-1} - 4a_{n-2}$. Proved that $a_n = 2^n(1 - \frac{n}{2})$ for all $n \geq 1$. (20 points)
4. Fifty cars sit on a parking lot. Twenty have stereo systems, 25 have air conditioners and 15 have sun roofs. Fifteen of the cars have at least two of these three options and 5 have all three. How many cars on the lot have *none* of these options? (20 points)
5. Prove that in any list of 21 integers, $a_1, a_2, \ldots, a_{21}$, there is s string of consecutive items of the list, $a_i, a_{i+1}, \ldots, a_j$ whose sum is divisible by 21. (20 points)