MATH 121 Sample Final Exam - Fall, 2006

Directions

- Answer all ten questions.
- Show your work neatly.
- You are **not allowed** to use any notes or index cards.
- Approximations will not be accepted, where *exact* answers are required.
  - Example: $1.41421\ldots$ will not be accepted in place of the *exact* value $\sqrt{2}$.
- Cell phones or any other communication devices must be turned off during the exam.

You may need the following formulas:

1. $\cos(x+y) = \cos x \cos y - \sin x \sin y$
2. $\sin(x+y) = \sin x \cos y + \cos x \sin y$
3. $a^2 = b^2 + c^2 - 2bc \cos A$
4. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Grading

#	points	score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total	200	

1. Determine the **exact** solutions (real and complex) of the equation:

$$x^5 - x^4 - x^3 - x^2 - 2x = 0.$$

2. Use logarithms to find the **exact** solution to $3^x = 4^{2x-1}$.

3. (a) Prove that $\dfrac{1}{\csc x - \sin x} = \sec x \tan x$.

   (b) Use identities to show that $\cos(x - \pi) = -\cos x$.

   (c) Find the **exact** value of $\cos \dfrac{7\pi}{12}$.

4. Let $a = 5$ cm, $b = 8$ cm, and $c = 9$ cm be the sides of triangle ABC. Determine the angles A, B, and C of the triangle. (Write your answers in degrees and accurate to two decimal places.)

5. Consider the function $f(x) = \dfrac{3x - 2}{x + 1}$.

   (a) Find the **inverse** of $f(x)$.

   (b) Find all asymptotes (vertical and horizontal) of $f(x)$.

   (c) Find the domain of $f(x)$.

6. Let $\cos x = \dfrac{3}{4}$ and $\dfrac{3\pi}{2} < x < 2\pi$. Find the **exact** values of

   (a) $\sin x$,

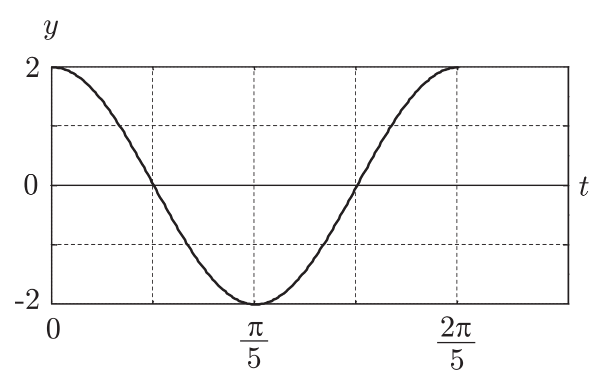
   (b) $\sin 2x$,

   (c) $\cot(-x)$.

7. (a) Find the amplitude, period, and phase shift of the function

$$g(t) = -3\cos\left(\frac{2\pi}{9}t - \sqrt{2}\right).$$

   (b) Find a possible formula of the form $A\sin(bt + c)$ or $A\cos(bt + c)$ for the function whose graph is shown below:



8. Consider the function $f(x) = 2x^2 + x + 1$.

   (a) Determine the equation of the function $g(x)$ obtained by performing the following transformations on $f(x)$:

      - expand by a factor of two,
      - shift one unit to the right,
      - shift 3 units upward.

   (b) Find the interval(s) for which $f(x)$ is increasing.

   (c) Find the interval(s) for which $f(x)$ is decreasing.

9. Let $z = 2 + i$.

   (a) Plot z in the complex plane.

   (b) Compute the modulus (absolute value) of z.

   (c) Write z in the polar form: $r(\cos\theta + i\sin\theta)$ where $0 < \theta < 2\pi$. (Write θ in radians and accurate to 3 decimal places.)

   (d) Compute z^8 and write your answer in the form $a + bi$.

10. Let $\mathbf{u} = \langle -1, 2\rangle$ and $\mathbf{v} = \langle 0, 3\rangle$.

   (a) Compute $\mathbf{v} - 2\mathbf{u}$.

   (b) Find the magnitudes (lengths) of $\mathbf{u}$ and $\mathbf{v}$.

   (c) Find a unit vector in the direction of $\mathbf{u}$.

   (d) Determine the direction angle of $\mathbf{v}$.

11. The mass $M(t)$ of a radioactive element at time t (measured in years) is given by

$$M(t) = c\left(\frac{1}{2}\right)^{t/h}$$

   where c is the mass at $t = 0$ and h is the half-life.

   (a) If $c = 250$ milligrams and it is known that $M(1000) = 100$ milligrams, determine the half-life h. (Write your answer accurate to two decimal places.)

   (b) Using the result from (a), determine the value of t when there is 1 milligram left. (Write your answer accurate to two decimal places.)