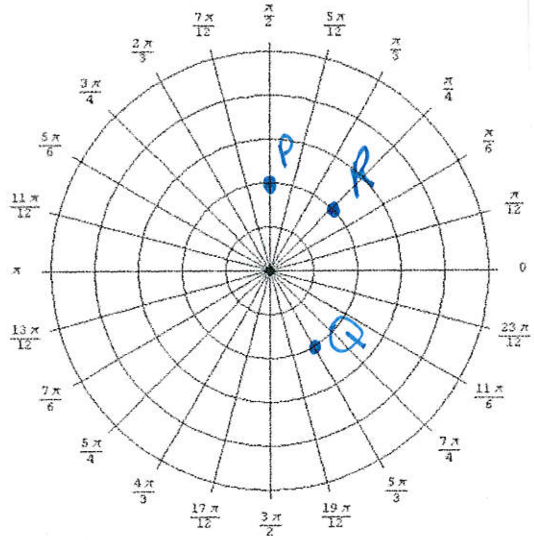
M121 W15 TU Review Solutions

1.



$(r\cos\theta, r\sin\theta)$

For P: $(2\cos\pi/2, 2\sin\frac{\pi}{2})$

$P(0, 2)$

For Q: $(2\cos(\frac{-\pi}{3}), 2\sin(\frac{-\pi}{3}))$

$Q(1, -\sqrt{3})$

For R: $(-2\cos(\frac{5\pi}{4}), -2\sin\frac{5\pi}{4})$

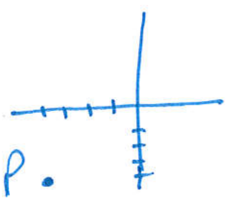
$(\sqrt{2}, \sqrt{2})$

2. a) note $\frac{5\pi}{6}$ is on same line as $-\frac{7\pi}{6}$, so $(3, -\frac{7\pi}{6})$

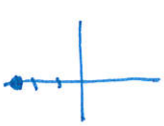
b) For r to be negative, we need to land on same line as $\frac{5\pi}{6}$, but in the opposite quadrant, so we need $\frac{11\pi}{6}$ $(-3, \frac{11\pi}{6})$

3. For P: $r = \sqrt{(-4)^2+(-4)^2} = \sqrt{32} = 4\sqrt{2}$ $\theta = \tan^{-1}(\frac{-4}{-4}) = \tan^{-1}(1) = \frac{3\pi}{4}$ (But 3rd Quad)

$P(4\sqrt{2}, 3\pi/4)$



For Q:



Notice this is on the x-axis, so $\theta = \pi$ & $r = 3$

$Q(3, \pi)$ or $Q(-3, 0)$

For R: $r = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$ $\theta = \tan^{-1}(\frac{3}{2})$

$R(\sqrt{13}, \tan^{-1}(\frac{3}{2}))$

$\rightarrow$

4. $\boxed{\begin{matrix} r\cos\theta = 4 \\ x = 4 \end{matrix}}$

$r = 3$

$\sqrt{x^2+y^2} = 3$

so $x^2+y^2 = 9$

$\theta = \pi/3$

$\tan\theta = \tan\pi/3$

$\frac{y}{x} = \sqrt{3}$

OR

$y = \sqrt{3}x$

$r = \frac{1}{\sin\theta+\cos\theta}$

$r\sin\theta + r\cos\theta = 1$

$y + x = 1$

OR

$y = 1 - x$

5a. $z_1 = -1 + i$

$\|z_1\| = \sqrt{(-1)^2+1^2} = \sqrt{2}$ $\qquad z_1 = \|z_1\|(\cos\theta + i\sin\theta)$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right)$ $\quad \theta = \frac{3\pi}{4}$ $\qquad z_1 = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

in 2^{nd} quad

$z_2 = 1 + \sqrt{3}i$ $\qquad$ in Quad I $\qquad \|z_2\| = \sqrt{1+3} = 2$ $\quad \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$z_2 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

b. $z_1 \cdot z_2 = 2\sqrt{2}\left(\cos\left(\frac{3\pi}{4}+\frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)\right) = 2\sqrt{2}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)$

$\frac{z_1}{z_2} = \frac{\sqrt{2}}{2}\left(\cos\left(\frac{3\pi}{4}-\frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)\right) = \frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

6. a. $\langle -4, -6\rangle + \langle 9, -12\rangle = \langle 5, -18\rangle$

b) $\|\vec{v}\| = \sqrt{2^2+3^2} = \sqrt{13}$ $\qquad \|\vec{w}\| = \sqrt{3^2+(-4)^2} = \sqrt{25} = 5$

c) $\vec{u} = \frac{\vec{v}}{\|v\|} = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$

d) $Q(x,y)$ then $\langle x-(-1), y-1\rangle = \langle 2, 3\rangle$ so

$x+1 = 2 \quad y-1 = 3$

$Q(1,4)$ $\quad x = 1 \quad y = 4$

e) If $R(x,y)$ then $\langle -2-x, 0-y \rangle = \langle 3,-4 \rangle$

so $-2-x=3$ and $0-y=-4$

$-x=5$ $\qquad$ $-y=-4$

$x=-5$ $\qquad$ $y=4$ $\qquad$ $R(-5,4)$

7. $\vec{v} = 3\vec{i} - 4\vec{j}$ & $\vec{w} = \vec{i} + 7\vec{j}$

same as $\vec{v} = \langle 3,-4 \rangle$ $\quad$ $\vec{w} = \langle 1,7 \rangle$

a. $\|\vec{v}\| = \sqrt{3^2+(-4)^2} = \sqrt{25} = 5$ $\qquad$ $\|\vec{w}\| = \sqrt{1^2+7^2} = \sqrt{50} = 5\sqrt{2}$

b. $\vec{v} \cdot \vec{w} = 3 \cdot 1 + -4 \cdot 7 = 3 - 28 = -25$

c. $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|} = \dfrac{-25}{5 \cdot 5\sqrt{2}} = \dfrac{-25}{25\sqrt{2}} = \dfrac{-1}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

so $\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

8) $\vec{F_1} = 600(\cos 30^\circ \vec{i} + \sin 30^\circ \vec{j}) = 300\sqrt{3}\,\vec{i} + 300\vec{j}$

$\vec{F_2} = 400(\cos 135^\circ \vec{i} + \sin 135^\circ \vec{j}) = -200\sqrt{2}\,\vec{i} + 200\sqrt{2}\,\vec{j}$

$\vec{F_1} + \vec{F_2} = (300\sqrt{3} - 200\sqrt{2})\vec{i} + (300 + 200\sqrt{2})\vec{j}$