1. Analyze the function and then sketch its graph.

- Find x- and y-intercepts of the graph.
- Determine the behavior of the graph near x-intercepts.
- Find the vertical asymptotes of the graph.
- Determine the behavior of the graph near its vertical asymptotes.
- Determine the intervals on which the function is positive/negative.
- Determine the end behavior of the graph.
- Find the horizontal/oblique asymptote.

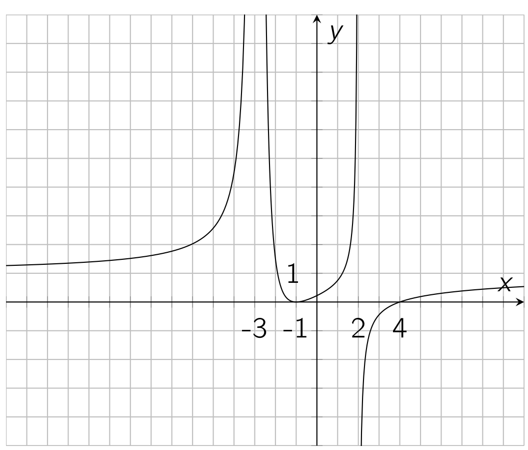
$$p(x) = (2x+1)(x-3)^2; \qquad g(x) = x^2(x-4)(x+1); \qquad R(x) = \frac{(x-1)(x+2)(x-3)}{x(x-4)^2};$$

$$Q(x) = \frac{3x^2-3x}{x^2+x-12}; \qquad H(x) = \frac{x^2-3x-4}{x+2}$$

2. Find the vertical, horizontal, and oblique asymptotes of the rational function.

$$F(x) = \frac{x+3}{(x-1)^2}; \qquad G(x) = \frac{x^2-9}{x^2+4x-21}; \qquad H(x) = \frac{x^3+1}{x^2-5x-14}; \qquad R(x) = \frac{x^4+1}{x}$$

3. The graph of a rational function $R(x)$ is shown below.



(a) Determine the domain of $R(x)$.
(b) Determine the range of $R(x)$.
(c) Give a possible formula for $R(x)$.

4. Suppose that $f(x) = (1/2)^{3x-1} - 4$. What is $f(2/3)$, $f(-1)$? If $f(a) = 508$ find a. Find zeros of $f(x)$.

5. Suppose that $g(x) = \log_3(x^2 - 16)$. What is the domain of $g(x)$? What is $g(5)$, $g(-5)$? Is $g(x)$ a 1-1 function? Find zeros of $g(x)$.

6. Use transformations to sketch the graphs of $f(x) = e^{x-2} - 4$, $g(x) = e^{-x} + 2$, $h(x) = -e^x + 3$, $p(x) = \ln(x-3) + 2$, $q(x) = -\ln(x+2)$, $r(x) = \ln(4-x)$.

7. Assume that $x > 0$ and write $\log_6\left(\dfrac{3(x+2)^2\sqrt[3]{x+4}}{7(x+3)^3x^{7/2}}\right)$ as a sum/difference of logarithms.

8. Write $2\ln(x^2 - 7x - 8) - \ln(x^3 + x^2) + 3\ln(x^3 + x)$ as a single logarithm. Simplify your answer.

9. Assume $\log_a 18 = 1.8$ and $\log_a 8 = 1.3$, find $\log_a(64)$, $\log_a(9/4)$, $\log_a(3\sqrt{2})$, $\log_a(1/18)$.

10. Solve the equation.

$$4^{x+2}2^x = 64; \qquad 3^{x+1} = 7^{1-2x}; \qquad \log_3(x-1)^2 = 2; \qquad \log_2(x+2) + \log_2(x+5) = 2$$

11. How long does it take your investment to triple if 6% interest is compounded continuously? How long will it take if 6% interest is compounded quarterly?

12. In a town whose population is 3000, a disease creates an epidemic. The number of people, N, infected t days after the disease has begun is given by the function

$$N(t) = \frac{3000}{1 + 14 \cdot 2^{-.2t}}$$

(a) Initially how many people are infected?
(b) Find the number of infected people after 5 days.
(c) After how many days is the number of infected people equal to 1600?
(c) In a long run, how many people will be infected?

1. $p(x) = (2x+1)(x-3)^2$:

- x-intercepts : $(-1/2, 0)$, $(3, 0)$; y-intercept: $(0, 9)$
- at $x = -1/2$ passes through axis: near $x = -1/2$, $p(x)$ is negative on the left ($x < -1/2$) and positive on the right ($x > -1/2$)
  at $x = 3$ bounces off axis: near $x = 3$, $p(x)$ is positive on both sides
- no VA, HA, OA
- positive on $(-1/2, 3) \cup (3, \infty)$, negative on $(-\infty, -1/2)$
- as $x \to \infty$ $y \to \infty$; as $x \to -\infty$ $y \to -\infty$

$g(x) = x^2(x-4)(x+1)$

- x-intercepts : $(-1, 0)$, $(0, 0)$, $(4, 0)$; y-intercept: $(0, 0)$
- at $x = -1$ passes through axis: near $x = -1$, $p(x)$ is positive on the left ($x < -1$) and negative on the right ($x > -1$)
  at $x = 0$ bounces off axis: near of $x = 0$, $g(x)$ is negative on both sides
  at $x = 4$ passes through axis: near $x = 4$, $g(x)$ is negative on the left ($x < 4$) and positive on the right ($x > 4$)
- no VA, HA, OA
- positive on $(-\infty, -1) \cup (4, \infty)$, negative on $(-1, 0) \cup (0, 4)$
- as $x \to \pm\infty$ $y \to \infty$

$$R(x) = \frac{(x-1)(x+2)(x-3)}{x(x-4)^2}$$

- x-intercepts : $(-2, 0)$, $(1, 0)$, $(3, 0)$; y-intercept: no
- at $x = -2$ passes through axis: near $x = -2$, $R(x)$ is positive on the left ($x < -2$) and negative on the lright ($x > -2$)
  at $x = 1$ passes through axis: $p(x)$ is positive on the left ($x < 1$) and negative on the right ($x > 1$)
  at $x = 3$ passes through axis: near $x = 3$, $R(x)$ is negative on the left ($x < 3$) and positive on the right ($x > 3$)
- VA: $x = 0$ and $x = 4$
- splits at $x = 0$: on the left ($x < 0$) $y \to -\infty$ and on the right ($x > 0$) $y \to \infty$
  at $x = 4$, $y \to \infty$ on both sides
- positive on $(-\infty, -2) \cup (0, 1) \cup (3, 4) \cup (4, \infty)$, negative on $(-2, 0) \cup (1, 3)$
- HA: $y = 1$
- as $x \to \pm\infty$ the graph approaches its horizontal asymptote $y = 1$

$Q(x) = \frac{3x^2-3x}{x^2+x-12}$

- x-intercepts : $(0, 0)$, $(1, 0)$; y-intercept: $(0, 0)$
- at $x = 0$ passes through axis: near $x = 0$, the function is negative on the left ($x < 0$) and positive on the right ($x > 0$)
  at $x = 1$ passes through axis: near $x = 1$, the function is positive on the left ($x < 1$) and negative on the right ($x > 1$)
- VA: $x = -4$ and $x = 3$

- splits at $x = -4$: on the left ($x < -4$) $y \to \infty$ and on the right ($x > -4$) $y \to -\infty$
  splits at $x = 3$: on the left ($x < 3$) $y \to -\infty$ and on the right ($x > 3$) $y \to \infty$
- positive on $(-\infty, -4) \cup (0, 1) \cup (3, \infty)$, negative on $(-4, 0) \cup (1, 3)$
- HA: $y = 3$
- as $x \to \pm\infty$ the graph approaches its horizontal asymptote $y = 3$

$H(x) = \frac{x^2-3x-4}{x+2}$

- x-intercepts : $(-1, 0)$, $(4, 0)$; y-intercept: $(0, -2)$
- at $x = -1$ passes through axis: near $x = -1$, the function is negative on the left ($x < -1$) and positive on the right ($x > 0$)
  at $x = 1$ passes through axis: near $x = 1$, the function is positive on the left ($x < -11$) and negative on the right ($x > -1$)
- VA: $x = -2$
- splits at $x = -2$: on the left ($x < -2$) $y \to -\infty$ and on the right ($x > -2$) $y \to \infty$
- positive on $(-2, -1) \cup (4, \infty)$, negative on $(-\infty, -2) \cup (-1, 4)$
- OA: $y = x - 5$
- as $x \to \pm\infty$ the graph approaches its oblique asymptote $y = x - 5$

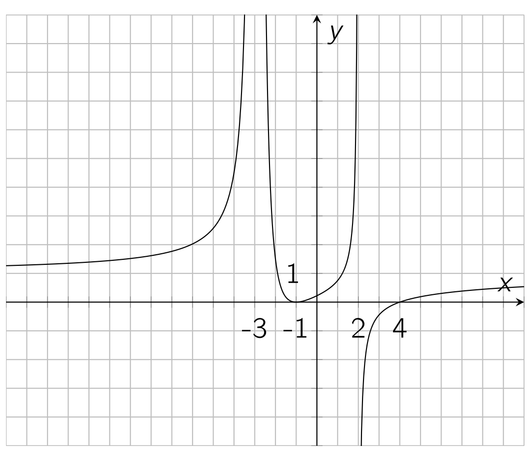
2. $F(x) = \dfrac{x+3}{(x-1)^2}$: VA $x = 1$; HA $y = 0$

$G(x) = \dfrac{x^2-9}{x^2+4x-21} = \dfrac{(x+3)(x-3)}{(x+7)(x-3)}$: VA $x = -7$; HA $y = 1$; hole at $x = 3$

$H(x) = \dfrac{x^3+1}{x^2-5x-14} = \dfrac{(x+1)(x^2+x+1)}{(x-7)(x+2)}$: VA $x = 7$ and $x = -2$; OA $y = x + 5$

$R(x) = \dfrac{x^4+1}{x}$: VA $x = 0$

3. The graph of a rational function $R(x)$ is shown below.



(a) domain of $R(x)$: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
(b) range of $R(x)$: $(-\infty, \infty)$
(c) possible formula for $R(x)$: $R(x) = \dfrac{(x+1)^2(x-4)}{(x+3)^2(x-2)}$

4. Suppose that $f(x) = (1/2)^{3x-1} - 4$.

$f(2/3) = -\frac{7}{2}$; $f(-1) = 12$

$f(a) = 508$ when $a = -\frac{8}{3}$; $f(a) = 0$ when $a = -\frac{1}{3}$

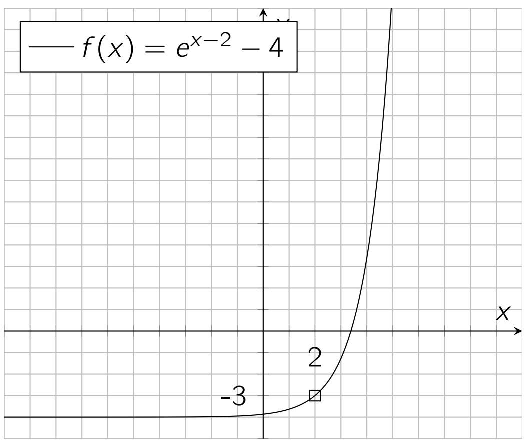
5. Suppose that $g(x) = \log_3(x^2 - 16)$.

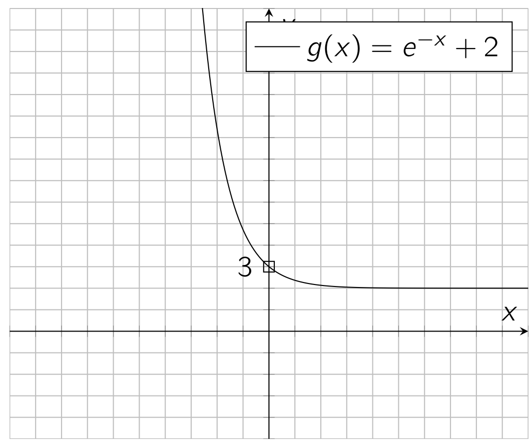
domain of $g(x)$: $(-\infty, -4) \cup (4, \infty)$

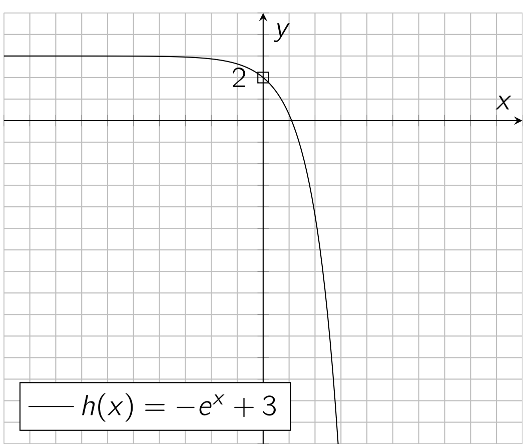
$g(5) = g(-5) = 2$, $g(x)$ is not a 1-1 function

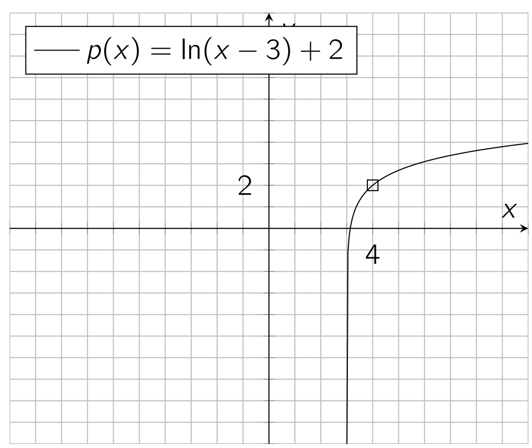
$g(a) = 0$ when $a = \pm\sqrt{17}$

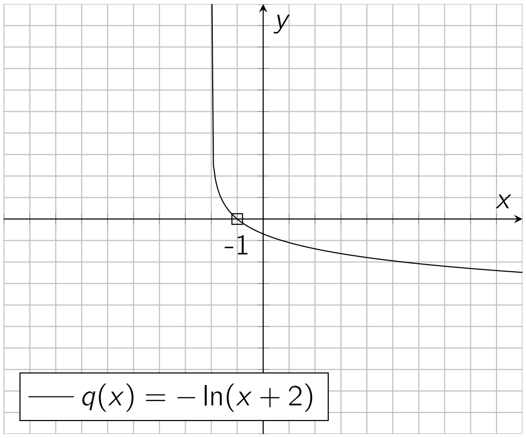
6. .

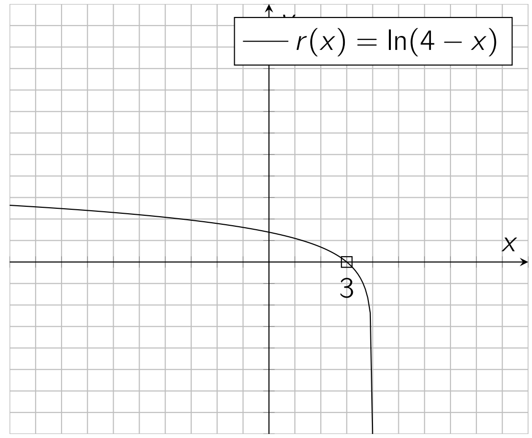












7. Assume that $x > 0$, then

$$\log_6\left(\frac{3(x+2)^2\sqrt[3]{x+4}}{7(x+3)^3x^{7/2}}\right) = \log_6 3 + 2\log_6(x+2) + \frac{1}{3}\log_6(x+4) - \log_6 7 - 3\log_6(x+3) - \frac{7}{2}\log_6 x$$

8. $2\ln(x^2-7x-8) - \ln(x^3+x^2) + 3\ln(x^3+x) = \ln\left(\frac{(x^2-7x-8)^2(x^3+x)^3}{(x^3+x^2)}\right) =$
$= \ln(x(x+1)(x-8)^2(x^2+1)^3)$

9. $\log_a(64) = 2.6$, $\log_a(9/4) = .5$, $\log_a(3\sqrt{2}) = .9$, $\log_a(1/18) = -1.8$.

10. $4^{x+2}2^x = 64 : x = 2/3$
$3^{x+1} = 7^{1-2x} : x = \dfrac{\ln 7 - \ln 3}{2\ln 7 + \ln 3}$
$\log_3(x-1)^2 = 2 : x = 4$ or $x = -2$
$\log_2(x+2) + \log_2(x+5) = 2 : x = -1$

11. How long does it take your investment to triple if 6% interest is compounded continuously?: $\dfrac{\ln 3}{.06}$ years
How long will it take if 6% interest is compounded quarterly? : $\dfrac{\ln 3}{4\ln 1.015}$ years

12. In a town whose population is 3000, a disease creates an epidemic. The number of people, N, infected t days after the disease has begun is given by the function

$$N(t) = \frac{3000}{1 + 14\cdot 2^{-.2t}}$$

(a) Initially how many people are infected? : 200 people
(b) Find the number of infected people after 5 days. : 375 people
(c) After how many days is the number of infected people equal to 1600?: 20 days
(c) In a long run, how many people will be infected?: 3000 people