MATH 121 – Precalculus
Fall 2014

Exam 3B
November, 24

Name: _____

UIN: _____

T/TH class time: _____

- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, or any electronic device including calculators and cell phones.
- Show ALL your work. Unsupported answers will not receive credit.
- Always state a complete answer to the problem.

Sum and Difference Formulas:

(1) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

(2) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(3) $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double-angle Formulas:

(4) $\sin(2\alpha)=2\sin\alpha\cos\alpha$

(5) $\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha$

(6) $\tan(2\alpha)=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

Half-angle Formulas:

(7) $\sin^2(\alpha/2)=\dfrac{1-\cos\alpha}{2}$

(8) $\cos^2(\alpha/2)=\dfrac{1+\cos\alpha}{2}$

(9) $\tan(\alpha/2)=\dfrac{1-\cos\alpha}{\sin\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}$

Product-to-Sum Formulas:

(10) $\sin\alpha\sin\beta=1/2[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

(11) $\cos\alpha\cos\beta=1/2[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$

(12) $\sin\alpha\cos\beta=1/2[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

Sum-to-Product Formulas:

(13) $\sin\alpha+\sin\beta=2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$ $\sin\alpha-\sin\beta=2\sin\left(\dfrac{\alpha-\beta}{2}\right)\cos\left(\dfrac{\alpha+\beta}{2}\right)$

(14) $\cos\alpha+\cos\beta=2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$ $\cos\alpha-\cos\beta=-2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

Law of sines:

(15) $\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{c}$

Law of cosines:

(16) $c^2=a^2+b^2-2ab\cos C;$ $b^2=a^2+c^2-2ac\cos B;$ $a^2=b^2+c^2-2bc\cos A$

Area of triangle:

(17) $K=\dfrac{1}{2}bh=\dfrac{1}{2}ab\sin C=\dfrac{1}{2}bc\sin A=\dfrac{1}{2}ac\sin B$

(18) $K=\sqrt{s(s-a)(s-b)(s-c)},$ where $s=\dfrac{1}{2}(a+b+c)$

(10 pts) **1.** Find the exact value:

$$\sin^{-1}\left(-\frac{1}{2}\right) = \qquad \cot^{-1}(\sqrt{3}) = \qquad \cos^{-1}\left(\cos\left(\frac{11\pi}{7}\right)\right) =$$

(10 pts) **2.** Write $\cot(\sin^{-1} u)$ as an algebraic expression in u.

(12 pts) **3.** Solve the equation on the interval $[0, 2\pi)$:

a) $\sin(\alpha) = \frac{1}{5}$

b) $\cot(\beta) = 9$

(10 pts) **4.** Consider the triangle with $A = 60°$, $b = 2$, $c = 5$.

a) Find side a.

b) Find the area of the triangle.

(10 pts) **5.** Consider the triangle with $a = 7$, $A = 30°$, $C = 30°$. Find side b only.

(11 pts) **6.** Given $\cos\theta = -\frac{1}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact value of:

a) $\sin\theta =$

b) $\sin(2\theta) =$

c) $\cos\left(\frac{\theta}{2}\right) =$

(9 pts) **7.** A building 300 feet tall casts a 110 foot long shadow. If you are looking from the top of the building to the tip of the shadow what angle does your line of sight makes with the vertical side of the building. Draw a picture supporting your solution.

(12 pts) **8.** Find all solutions to the equation:

$$\cos\left(3\theta + \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$$

(16 pts) **9.** Establish the identity: start with one side and derive the other side.

a) $\cos(\pi + \alpha) = -\cos\alpha$

b) $\dfrac{\cos\alpha + \cos\beta}{\sin\alpha + \sin\beta} = \cot\left(\frac{\alpha+\beta}{2}\right)$