Math 125 Final Exam Practice

## CHAPTER 1:

1.1: List the Intercepts of each Equation and then sketch the graph

a) 18x + 10y = 90 b) 16x + 24y = 432 c) 25x + 10y = 500

1.2: USING ALGEBRA(meaning no calculators), find the Intersection of the two Lines.

a) 
$$4x-5y=16$$
  
 $6x+y=7$   
b)  $4x+5y=25$   
 $6x+2y=21$ 

1.3: The Tablette Corporation makes tablet computers at plants in Thailand and Malaysia. At the Thailand plant, unit costs are \$79 and fixed costs are \$9000, while in Malaysia unit costs are \$71 and fixed costs are \$9400.

a) If Tablette needs 600 tablets made, which plant does so at a lesser cost? JUSTIFY YOUR ANSWER

b) What number of tablets made will have the same total cost at both facilities?

1.4: A home appliance manufacturer has been selling a kitchen stove model several markets and wishes to enter new markets. Atlanta has 6.1 million people and they sold 13,286 stoves, Tampa has 2.8 million with 5,123 sold, Miami has 6.4 million with 17,522 sold, Charlotte(NC) has 2.5 million with 4,848 sold, and Greenville(SC) has 1.4 million with 3,613 sold.

a) Using Population figures in Millions and sales as they are given, use LinReg in constructing the Least Squares Line of Best Fit for this data set. **Round-off values to the nearest thousandth**.

b) Use your line to predict the sales generated by entering a market like Orlando with 2.9 million people.c) What sized market does your model suggest is necessary to sell 20,000 units?

1.5: The table below shows sales data for Lectrik Motors, with sales in 2014 of their electric vehicles as a function of the population of various metropolitan areas(in 1000's).

0.1

1. 1.4

a) What is the Line of Best Fit, in $Y = AX + B$ form? Use your calculator
Round values to 2 places behind the decimal
b) What is the expected number of cars sold in an area with a population of 950000
people? SHOW WORK using your answer to Part (a). Round to the nearest Whole
Number
a) What size site would it take to see sales of 500 webicles? A narrow in 1000's

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c) What size city would it take to see sales of 500 vehicles? **Answer in 1000's**. **SHOW WORK** using your answer to Part (a). Round to the nearest Whole Number

Sales
96
162
83
254
188
330
147
221

### CHAPTER 2:

2.1: Solve each System of Equations using the "RREF" function on your calculator. Show your augmented matrix and the resulting one from RREF, plus your solution.

$$-x-2y+3z = -14$$
 $x+y-z = 15$  $-4x-y+z = -14$ a)  $x+y-8z = 10$ b)  $2x+3y-2z = 27$ c)  $2x+y+z = 6$  $x-y-18z = 3$  $-2x-3y+3z = -30$  $5x+y-2z = 18$ 

2.2: Write each Elementary Row Operation in notational values and then perform it on the matrix, using the original Matrix each time.

3	2	-2	-6	5 ]
-3	12	7	-2	-6
0	-4	5	1	3
1	-2	-3	-7	5 ]

- a) Multiply Row 4 by positive three
- b) Add minus 2 times row four to row one
- c) Add 5 times row one to row three
- 2.3: Given the following matrices, answer the questions below. You may use a calculator.

$$A = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix} B = \begin{bmatrix} 3 & 8 & 1 \\ 9 & -2 & -6 \\ -5 & 7 & 3 \end{bmatrix} C = \begin{bmatrix} 2 & -5 & -6 \\ -1 & 4 & 9 \end{bmatrix} D = \begin{bmatrix} 7 & -2 \\ 8 & 1 \\ -5 & -6 \end{bmatrix} E = \begin{bmatrix} -4 & 8 & -1 \\ -5 & 3 & 7 \end{bmatrix}$$

a) Find: 2C - 3E

b) Find the product DE.

c) Is BDECA possible? Show why or why not

	2	-3	4	1 ]
2.4. Direct on the element in Dorry 2 and Column 2 DV HAND	0	5	-2	7
2.4: Pivot on the element in Row 3 and Column 2, BY HAND:	-8	2	4	6
2.4: Pivot on the element in Row 3 and Column 2, BY HAND:	1	4	6	-4

2.5: If 
$$A = \begin{bmatrix} 13 & -18 \\ 6 & 17 \end{bmatrix}$$
 and  $B = \begin{bmatrix} -10 & 7 \\ 16 & -23 \end{bmatrix}$ , and  $2A - C = B$ , find the matrix C.

You may use a calculator, but show any commands used.

2.6: A certain economy has three industries: metals, plastic, and wood. To produce \$1 of output, the metals industry needs \$0.08 of its own output, \$0.03 from plastics, and \$0.07 from wood. To produce \$1 of output, the plastic industry needs \$0.11 of its own output and \$0.02 from wood. To produce \$1 of output, the wood industry needs \$0.10 of its own output, \$.05 from metals, and \$0.04 from plastic. If the population needs(in Millions) \$32 of metal, \$25 of plastic, and \$40 of wood, how much of each must be produced to satisfy this demand?

a) Name and show ALL relevant matrices

b) Use  $X = (I - A)^{-1} D$  (calculator allowed) to determine production levels, rounded to the nearest tenth of a Million.

2.7: The Matrices below are Inverses of one another. Show ALL WORK in solving the system that follows,

USING the  $X = A^{-1}B$  method.

Γ	2	2	1		[-2	1	3	-2x + y + 3z = 4
-	-1	1	-1	and	1	0	-1	x - z = -2
L	2	1	1	and	3	-2	-4	3x - 2y - 4z = 8

2.8:  
$$3x + 2y - 2z - 2w = -2$$
$$2x + 2y - 4z - 2w = 3$$
$$-x - 2z + w = 2$$

-x - 2y + 6z = -2

For the system of linear equations given above, find the value of X in a solution where Y = -2.5

2.9: A small economy has three industries: coal, steel, and electricity. To produce \$1 of coal, it takes \$0.04 of coal, \$0.20 of steel, and \$.08 of electricity, while to produce \$1 of steel, it takes \$0.15 of coal and \$0.10 of electricity. To produce \$1 of electricity, it takes \$0.02 of their own output, plus \$0.30 of coal and \$0.10 of steel. If the economy needs \$54 million of coal, \$26 of steel, and \$85 of electricity, how much must be produced in the electricity industry, in millions to the nearest tenth?

(Hint: the useful formulas are X - AX = D  $(I - A)^{-1}D = X$ )

2.10: Given 
$$A = \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$$
,  $C = \begin{bmatrix} -6 & -2 \\ 11 & 6 \end{bmatrix}$  and  $AB = C$ , find the total:  $b_{2,1} + b_{2,2}$ 

### CHAPTER 3:

3.1: Solve the Linear Programming Problem.

Graph the Inequalities and determine the Feasible Set. SHOW WORK. Clearly mark your Axes and label graphed points with coordinates for full credit.

 Maximize : 8x + 3y s.t.
 Minimize : 600 - 5x + 7y s.t.

  $5x + y \le 360$   $2x - 3y \le 0$  

 a)  $x + 3y \le 180$  b)  $2x - 3y \le 0$ 
 $x + y \le 80$   $x \ge 0, y \ge 0$ 

3.2: DigginDeep Mining company has to fill an order for 40 tons of iron ore and 18 tons of copper ore. They operate two mines producing both ores. Mine I yields four tons of iron and one ton of copper and costs \$3200 per day to operate, while mine II yields two tons of iron and two tons of copper and costs \$2600 per day to operate. Determine how many days each mine should be used to fill this order at the least total cost.

SET UP(do NOT solve) a Linear Programming Problem for this situation.

3.3: Graph, BY HAND, the Feasible Set of the system below.

 $x + 2y \le 36$  $x + y \le 20$  $x \ge 0, \ y \ge 4$ 

3.4: A plane carries two types of packages. Type A weighs 140 pounds and takes up 3 cubic feet of space and generates \$175 in revenue, while a Type B package weighs 185 pounds and takes up 5 cubic feet, generating revenues of \$240 per package. The plane is limited to 6000 total pounds in 300 total cubic feet of space. Construct a full Linear Programming problem with an Objective function and ALL necessary constraints. DO NOT SOLVE.

3.5: Use the graphing methods of Chapter 3 to solve the Linear Programming problem below.

	$Minimize: 900+5x+7y \ s.t.$	Maximize: $15x+12y$ s.t.
	$2x + 4y \ge 64$	$x + y \le 65$
a)	$6x + 2y \ge 72$	b) $x + 3y \le 135$
	$x + 4y \ge 44$	$5x + 2y \le 250$
	$x \ge 0, y \ge 0$	$x \ge 0, y \ge 0$

3.6: A new wildlife refuge is being populated with animals, specifically lions and tigers and bears. The park manager knows he can fit exactly 150 animals total. His experience tells him to have at least twice as many lions as tigers. Area limitations suggest he acquire no more bears than tigers. To make sure the public is likely to see each species during their visit, he wants at least 25 of each. Lions cost \$40/day to care for, tigers cost \$30, and bears cost \$35. Help the manager build a population meeting his needs at the lowest daily cost.

# Set up the Linear Programming Problem using only TWO variables. DO NOT SOLVE.

3.7: ListenUp company makes speaker cabinets in Chicago, with 400 on hand, and Detroit, with 600 on hand, and has showrooms in Milwaukee, where they need 250, and Cincinnati, where they need 280. It costs \$14 to ship each cabinet from Chicago to Milwaukee, \$22 from Chicago to Cincinnati, \$20 from Detroit to Milwaukee, and \$18 from Detroit to Cincinnati. Help ListenUp get the cabinets to the showrooms at the least total shipping cost.

Build an appropriate LP-Problem, but Do Not Solve. USE X = # shipped from Detroit to Milwaukee and Y = #shipped from Detroit to Cincinnati.

## CHAPTER 4:

4.1: Furniture Factory produces tables, chairs, and desks. Each table needs 3 hours of carpentry, 1 hour of sanding, and 2 hours of staining, with a profit of \$12.50. A chair needs 2 hours of carpentry, 4 hours of sanding, and 1 hour of staining, with a profit of \$20. Desks need 1 hour of carpentry, 2 hours of sanding, and 3 hours of staining, with a profit of \$16. There are 660 hours of carpentry, 740 hours of sanding, and 853 hours of staining available each week. The manager would like to make as much money as he can.

a) Construct the Linear Programming problem, including an Objective Function and ALL constraints

b) Build a fully labeled Initial Simplex Tableau. Indicate where and why you will pivot.

c) Pivot until you reach an Optimal Solution, and **show the final Tableau**. You may **skip** showing any intermediate Tableaux

d) State your solution in terms of the word problem above

4.2: Simplex Method and Duality. Solve using the Method of Duality

Minimize: 10x + 15y subject to  $x + y \ge 120$   $3x + y \ge 240$   $2x + 4y \ge 480$  $x \ge 0, y \ge 0$ 

### 4.3: A Linear Programming problem and its Optimal Tableau are given below:

Maximize: 3x + 5y + 2z  s.t.	$\int x$	у	Z.	и	v	W	М	]
$2x + 4y + 2z \le 34$	$\overline{0}$	1	0	-5/2	1	1	0	2
$3x + 6y + 4z \le 57$	0	-1	1	1	0	-1	0	4
$2x + 5y + z \le 30$	1	3	0	-1/2	0	1	0	13
$x \ge 0$ , $y \ge 0$ , $z \ge 0$	0	2	0	1/2	0	1	1	47

a) If the RHS of Constraint #1 were to be changed to 30, determine the New Optimal M-value and the value of each decision variable. USE sensitivity analysis, NOT by redoing Simplex Method

b) If the RHS of Constraint #3 were to be changed to 32, determine the New Optimal M-value and the value of each decision variable. USE sensitivity analysis, NOT by redoing Simplex Method

c) If the RHS of Constraint #1 were changed to 34 + h, find the range of h so that we are still feasible.

4.4: Solve by use of Simplex Method  $Maximize: 11x+8y \quad s.t.$ 

 $2x + 3y \le 60$ 

 $4x + y \le 40$ 

 $x + y \le 30$ 

 $x \ge 0$  ,  $y \ge 0$ 

4.5: Simplex Method and Non-Standard Pivots.

a) Convert the problem into "Standard Max" Form

b) Construct an Initial Tableau and choose a Pivot Element

c) Pivot until Optimal and list the Solution to the original LP-Problem

Minimize: 10x + 15y subject to  $x + y \ge 120$   $3x + y \ge 240$   $2x + 4y \ge 480$  $x \ge 0$ ,  $y \ge 0$ 

## CHAPTER 8:

8.1: Business Math students are used in an experiment, where they are sent into a T-shaped maze once each day. If they turn left to exit the maze, they are given a slice of apple pie, while if they turn right to exit the maze, they are given a zap of electricity. It has been observed that the day after they turn left(apple pie!), they will turn left again the next day 55% of the time, and that the day after they turn right, they will turn right again the next day 75% of the time.

a) Build and label a transition matrix and initial distribution matrix

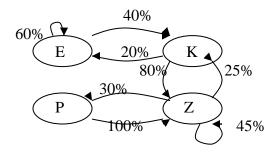
b) If on the first day, half of the students go each way, what percent will go right the next day?c) USE ALGEBRA to determine the probability of students going each way in the long run. You may check with use of your calculators, but no credit will be given without an algebraic solution.

8.2: The initial absorbing matrix below has been converted to Standard form. Use  $\begin{bmatrix} I & S(I-R)^{-1} \\ 0 & 0 \end{bmatrix}$ 

to determine what percentage of the population starting in Y will eventually end up in Z.

	W	X	Y	Ζ			X	Ζ	W	Y
W	[.2	0	.15	0		X	1	0	.3	.45]
X	.3	1	.45	0	$\Rightarrow$	Ζ	0	1	.1	.35
Y	.4	0	.05	0		W	0	0	.2	.15
Ζ	[.1	0	.35	1						.05

8.3: Build a properly labeled Transition Matrix for the Transition Diagram below:



8.4: Shares of stock in a widely traded company have been observed to follow a noticeable pattern, in that the day after the stock price rises, it will rise again 50% of the next trading days, fall 10% of the time, and stay even the rest of the time. If the stock price falls one day, it will fall again 20% of the time, rise 45% of the time, and stay even the rest of the time. The stock price never stays even on consecutive days, rising 70% of the time after staying even.

a) Construct a fully labeled Transition Matrix

b) A stock trader knows the company will publish an earnings report on Monday and decides, based on his analysis, that the stock price has a 25% chance of rising and 75% of falling Monday. He wants to determine what probability is there of the stock price rising on Thursday. Build the matrix X, and show the necessary calculation to help answer his question.

c) If this stock price trend were to continue, what percent of days will the stock price rise, fall, or stay even in the long run? (Round % values to the nearest 0.1%).

#### CHAPTER 9:

9.1: Find the *strictly determined* solution of the game matrix below. Show your reasoning.

a) 
$$R\begin{bmatrix} 2 & 0 & 1\\ 0 & -3 & 4\\ 3 & -2 & 0 \end{bmatrix}$$
 b)  $R\begin{bmatrix} -2 & 3 & 4\\ -1 & 2 & 1 \end{bmatrix}$  c)  $R\begin{bmatrix} -6 & 1\\ 4 & 3\\ -5 & 2 \end{bmatrix}$ 

9.2: Player R has \$1, \$10, and \$50 bills, while player C has \$5 and \$20 bills. Each randomly chooses a bill and shows it for each play of the game. The one with the larger bill collects the difference between their bill and that of the other player.

Write the payoff matrix for this game, showing any necessary labels on the matrix. Decide whether this game is Strictly Determined or not. If it is strictly determined, what is the Value of the game?

9.3: Below is a payoff matrix for a certain game. Do the requested tasks which follow.

$$\begin{array}{c} C \\ R \begin{bmatrix} -15 & 40 & 25 \\ 25 & -10 & -5 \\ 45 & 20 & -15 \end{bmatrix}$$

a) Show that the matrix does NOT have a strictly determined solution.

b) Find the expected payout if  $R = \begin{bmatrix} .2 & .4 & .4 \end{bmatrix}$  and  $C = \begin{bmatrix} .5 \\ 0 \\ .5 \end{bmatrix}$ 

9.4: Adjust the Game Matrix below with the smallest possible Integer so that all values are positive. Construct either Linear Programming Problem which can be used to find the optimal play for each of R and C. Solve the LP problem. Find the fraction of the time each player should make their choices, and find the value of the game(in regards to the original matrix form)

$$\begin{array}{c} C \\ R \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix} \end{array}$$

9.5: Adjust the Game Matrix below with the smallest possible Integer so that all values are positive. Construct either Linear Programming Problem which can be used to find the optimal play for each of R and C. Solve the LP problem. Find the fraction of the time each player should make their choices, and find the value of the game(in regards to the original matrix form)

$$\begin{array}{c}
C \\
R \begin{bmatrix}
-2 & 3 & 0 \\
1 & -1 & 2 \\
0 & 4 & 1
\end{array}$$

9.6: Rick and Cathy are playing a game, each having 3 cards. Rick has Black 4, a Red 7, and a Black 9, while Cathy has a Red 2, a Black 6, and a Red 8. Each shows a card. If both cards are the same color, Cathy wins the difference of the two cards, but if they cards are different colors, Rick wins the amount on the smaller card.

a) Set up a fully labeled Payoff Matrix, and determine whether it has a Saddle Point.

b) Suppose Rick plays each of his cards equally often, while Cathy play her Red cards 40% of the time each. Set up appropriate matrices *R* and *C*, and then calculate the expected value under these circumstances(round to the nearest 0.01 as necessary).

c) Determine each players optimal play options and the expected value of the game when they make those respective plays percentages(round v to the nearest 0.01 as necessary).