

#### SECTION 4.1: BUILDING A SIMPLEX TABLEAU AND PROPER PIVOT SELECTION

*Maximize* :  $15x + 25y + 18z$  *s.t.*

$$2x + 3y + 4z \leq 60$$

$$4x + 4y + 2z \leq 100$$

$$8x + 5y \leq 80$$

$$x \geq 0, y \geq 0, z \geq 0$$

- a) Build Equations out of each of the constraints above by introducing Slack Variables
- b) Convert the Objective Function to an Equation by setting it = M. Move all other variables to the same side as M, and place below the Constraint/Slack Variable Equations
- c) Place the Coefficients from all Equations into a Simplex Tableau, labeled above with variables indicating their respective columns
- d) The first step here in choosing a Pivot element is to note the “most negative number” in the bottom row(except possibly the bottom right corner value for M). Which Column is that?
- e) Once a Column is selected, calculate the ratios of each Right-Hand side value over the pivot column value. The smallest Non-Negative Ratio indicates the Pivot Row. Which Row has the smallest Non-Negative value?

- f) Put the Initial Tableau into your calculator, and save a copy into another matrix  
Perform a Pivot on the position Row 1, Column 2 (this is not the Row we chose in part e)  
What do you notice has happened in the Right-Hand column? A negative number has appeared, which tells us we are “not feasible”, and we have performed an incorrect Pivot. This should never happen when we Pivot from a Feasible situation.
- g) Start the Pivot process over, going back to the Initial Tableau in part (c), and Pivot where we decided was the correct Pivot element in part (d) and (e). Write the new Tableau down in the space below.  
Notice there are no negative numbers in the Right-Hand column, so we are feasible.
- h) List the values of ALL variables, a “new” Current Solution: decision variables, slack variables, and M
- i) Determine where the next Pivot should occur, but **DO NOT PIVOT**.

EXERCISE: Given the LP-Problem

*Maximize* :  $16x + 22y$  s.t.

$$3x + y \leq 180$$

$$x + 4y \leq 260$$

$$x + y \leq 80$$

$$x \geq 0, y \geq 0$$

a) Convert the LP-Problem into equations with Slack Variables, also turning the Objective Function into a properly built equation in the space above right

b) Use the equations to build an Initial Simplex Tableau, with appropriate column labels


c) Indicate where you should Pivot in the above Tableau, showing the Ratios used. Then perform the Pivot on your calculator and write out the new Tableau in the space above on the right.

d) Verify that we are Feasible. Then, are we Optimal? Yes or No ? List the “new” Current Solution from this new Tableau

e) Again, indicate where the next Pivot should be done, Ratios shown fully. Perform that Pivot on your calculator and write out the new Tableau below.

f) Feasible? Yes or No      Optimal? Yes or No      New Current Solution?

## SECTION 4.2: USING SIMPLEX TO FIND AN OPTIMAL MAXIMUM SOLUTION

**EXERCISE:** Furniture Factory produces tables, chairs, and desks. Each table needs 3 hours of carpentry, 1 hour of sanding, 2 hours of staining, with a profit of \$12.50. A chair needs 2 hours of carpentry, 4 hours of sanding, 1 hour of staining, with a profit of \$20. Desks need 1 hour of carpentry, 2 hours of sanding, 3 hours of staining, with a profit of \$16. There are 660 hours of carpentry, 740 hours of sanding, and 853 hours of staining available each week. The manager would like to make as much money as he can.

- Construct the Linear Programming problem, including an Objective Function and ALL constraints
- Introduce Slack Variables into each Constraint, and introduce M to the Objective function by turning it into an equation. SHOW ALL of these Equations below
- Build a fully labeled **Initial** Simplex Tableau above right. Indicate where and why you will pivot.
- Pivot until you reach an Optimal Solution, and **show the final Tableau**. You may **skip** showing any intermediate Tableaux
- State your solution in terms of the word problem above

EXERCISE: Given the LP-Problem below, do the tasks that follow.

*Maximize* :  $4x + 6y + 3z$  *s.t.*

$$4x + 4y + 8z \leq 800$$

$$8x + 6y + 4z \leq 1800$$

$$2x + 4y + 6z \leq 600$$

$$x \geq 0, y \geq 0, z \leq 0$$

a) Introduce Slack Variables into each Constraint, and introduce M to the Objective function by turning it into an equation. Build a fully labeled **Initial** Simplex Tableau. Indicate where and why you will pivot.

b) Pivot until you reach an Optimal Solution, and **show the 2<sup>nd</sup> and Final/Optimal Tableau**.

c) State your solution, ALL Variables

EXERCISE: Given the LP-Problem below, perform the tasks which follow

*Maximize* :  $3x + 5y + 5z$  *s.t.*

$$x + y + z \leq 100$$

$$3x + 2y + 4z \leq 210$$

$$x + 2y \leq 150$$

$$x \geq 0, y \geq 0, z \leq 0$$

- a) Introduce Slack Variables into each Constraint, and introduce M to the Objective function by turning it into an equation. Build the Initial Simplex Tableau, and put it into your calculator.
- b) Decide where and why you will pivot. Following proper procedures, you should look for “the most negative value” in the bottom row. But here we have a tie, in -5 in both columns y and z. Which one should we use? It actually should not matter, so we will do it both ways and see what happens.
- c) Select the proper Row in Column y, then Pivot until you reach an Optimal Solution, and **show the final Tableau**. You may **skip** showing any intermediate Tableaux
- d) Go back the Initial Tableau and repeat the process for Column z, choosing the correct Pivot Row.
- e) Did you get the same Optimal Solution both times? What is it?

## SECTION 4.3: NON-STANDARD CONSTRAINTS AND MINIMUM OBJECTIVE FUNCTIONS

### NON-FEASIBLE STARTING SOLUTION

*Maximize* :  $4x + 6y$  *s.t.*

$$2x + y \geq 15$$

$$4x + 5y \leq 100$$

$$x + y \geq 10$$

$$x \geq 0, y \geq 0$$

a) Rewrite Constraints 1 and 3 to proper form

b) Introduce Slack Variables and Construct an initial Simplex Tableau

c) Suppose we choose to look at the Negative Number in the Right Column, Row 1, and then choose Column 2, the Y-column. Calculate non-negative ratios, which indicate a Pivot in Row 3, not our originally noted Row 1. Perform this Pivot and show the new Tableau. Are we Feasible? (no, not yet)

d) Continue Pivoting until reaching an Optimal Solution, SHOW the final tableau  
List the value of every variable(X,Y,U,V,W, and M)

e) Go back to the Initial tableau, and instead of Column 2(Y-var), select Column 1(X-var) for your first Pivot. Solve completely and compare Optimal Solution to the first time.

## SOLVING A MINIMUM PROBLEM BY CONVERTING TO A MAXIMUM

*Minimize* :  $18x + 36y$  *subject to*

$$3x + 2y \geq 24$$

$$5x + 4y \geq 46$$

$$4x + 9y \geq 60$$

$$x \geq 0, y \geq 0$$

a) Convert the Min to Max and fix ALL Constraints

b) Build the Initial Simplex Tableau, indicate where/why you will Pivot and perform the pivot  
Show BOTH the Initial and 2<sup>nd</sup> Tableaux

c) Finish the Pivoting process until Optimal, SHOW the final Tableau

d) List the values of X and Y, found in the bottom row of their respective columns  
Remember to Multiply the M-value by -1 for the proper solution to the Minimum



EXERCISES: Solve each of the following LP-Problems. Convert Constraints and/or Objective Functions to necessary form, as needed. Show each Initial Tableau, Pivot choice, and Final Tableau. State each Optimal Solution.

*Minimize* :  $30x + 15y + 28z$  *s.t.*

$$5x + 3y + 4z \geq 45$$

i)  $5x + 6y + 8z \geq 120$

$$10x + 3y + 7z \geq 150$$

$$x \geq 0, y \geq 0, z \geq 0$$

*Maximize* :  $10x + 50y + 30z$  *s.t.*

$$3x + 6y + 2z \leq 450$$

ii)  $5x + 16y + 8z \geq 120$

$$3x + y + z \geq 300$$

$$x \geq 0, y \geq 0, z \geq 0$$