SECTION 1.1: Plotting Coordinate Points on the X-Y Graph

This should be a review subject, as it was covered in the prerequisite coursework. But as a reminder, and for practice, plot each of the following Coordinate Pairs in (x, y) form by first moving in direction x and then in direction y the distance specified by each x or y value. The axes are not marked, so first place "tick-marks" along all 4 directions, evenly spaced, and then number them by 1's(1, 2,3… or -1, -2, -3…)

$(2,3)$

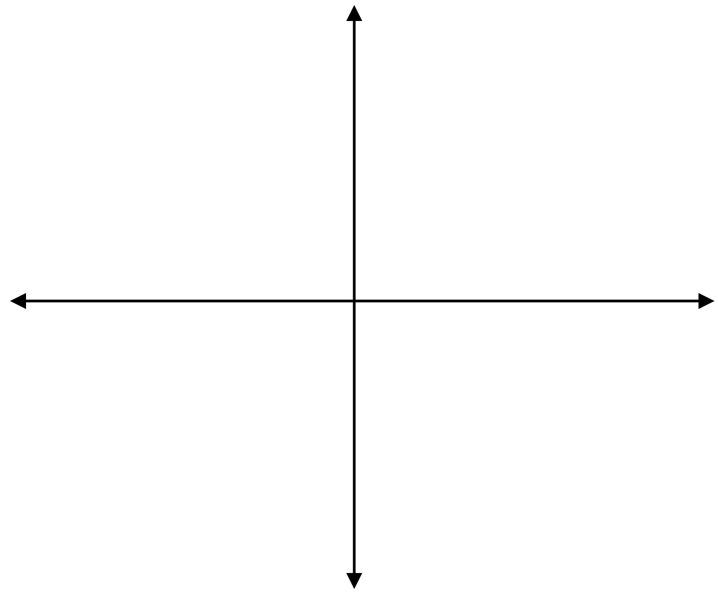
$(-1,2)$

$(0,1)$

$(4,0)$

$(-2,-3)$

$(3,-2)$



Now let's try some Coordinate Pairs that resemble ones you will definitely see later in this course, ones that stretch further out than 2 or 3 in either direction. Before jumping the gun, stop and think how you will mark the axes. By 1's like the above exercise? Not much use, right? So, make each tick-mark 10 apart from the next. You will have to place some values between successive marks, but that's OK.

$(25,0)$

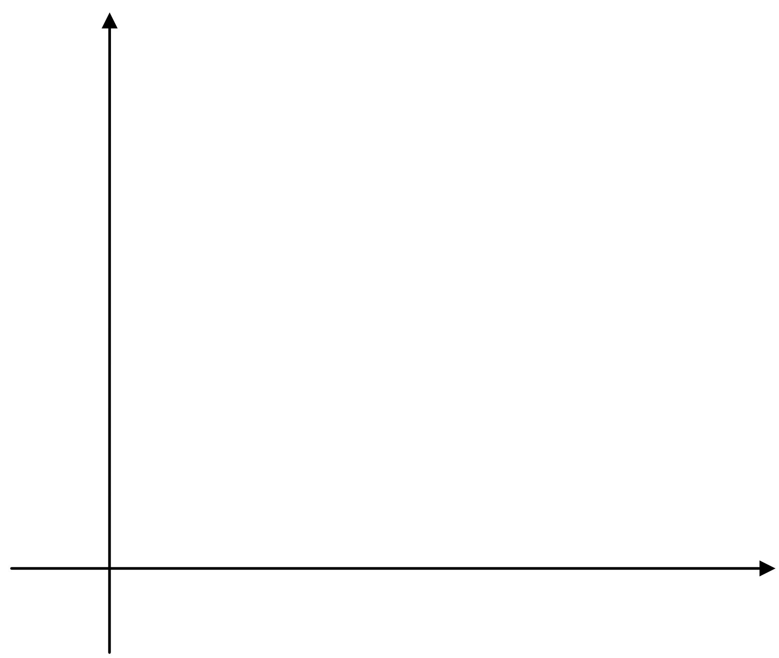
$(40,0)$

$(56,0)$

$(0,32)$

$(0,20)$

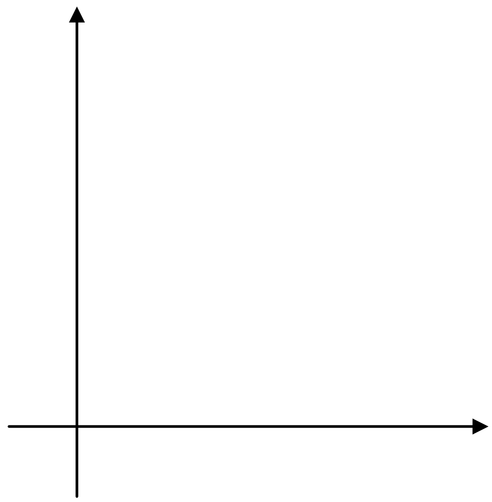
$(0,60)$



Note that ALL of these points are Intercepts(along an axis), something you should see often in later work.

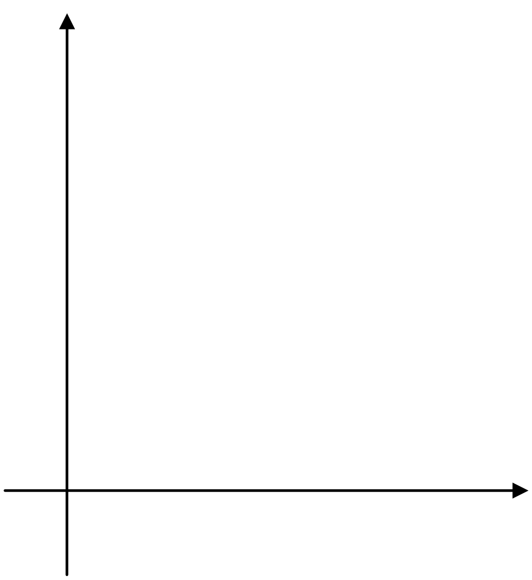
Now let's look at graphing lines. There are many forms the equation of a line, like "General", which has the look $cx + dy = e$ and Slope-Intercept, with the look $y = mx + b$. While the book and MyMathLab will tell you to turn the General form into Slope-Intercept form, I would prefer you use the following method, since it works best for the types of equations you will encounter most often later in this course.

Start with the equation $6x + 2y = 12$. There is really no need to convert to Slope-Intercept, when finding both Intercepts is relatively easy. It gives us two useful points to plot, plenty to sketch a decent graph. Start by plugging in a 0 for *y*, and solve for *x*, put in $(x, 0)$ form. Then plug in a 0 for *x*, and solve for *y*, and put in $(0, y)$. Appropriately mark both axes, with numbers, and list each coordinate pair by its plotted point. Then sketch a line through both.

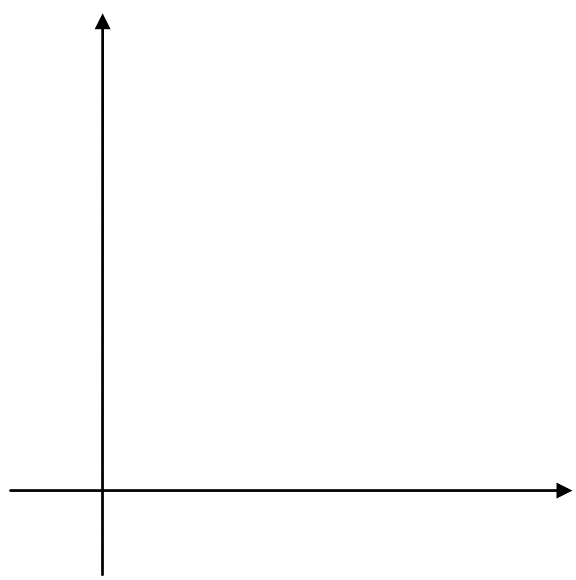


Now, do the same thing for each equation below. How will you mark/number each axis? By 1's? Hopefully not, but don't decide until you have the Intercepts. Should they be done the same? (No)

$12x + 6y = 168$



$10x + 15y = 900$



1.1 EXERCISES

1) Determine whether each coordinate pair is on the line $4x+7y=224$

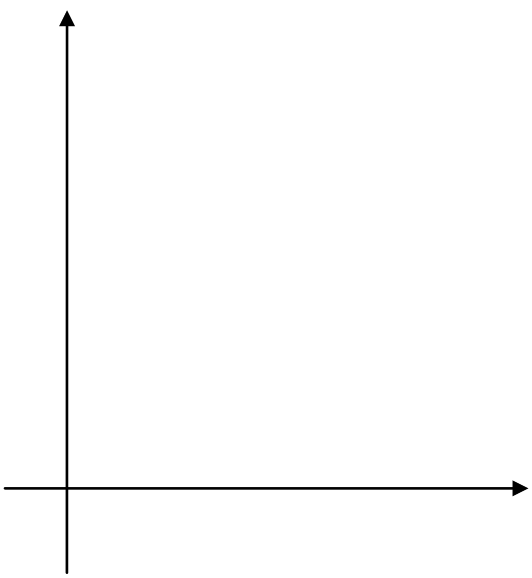
a) $(54,0)$ b) $(50,2)$ c) $(32,16)$ d) $(77/2,10)$ e) $(12,20)$ f) $(16,23)$

2) Find BOTH Intercepts, in Coordinate form, for each Linear Equation: Fractions only, NO Decimals

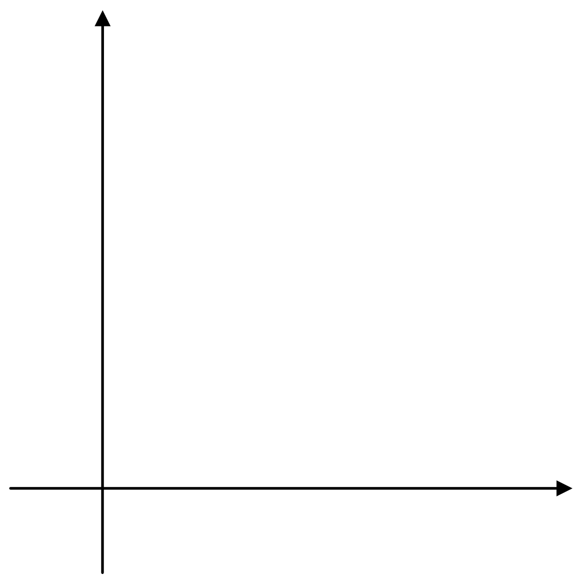
a) $24x+18y=864$ b) $20x+45y=1200$ c) $6x+10y=75$

3) Graph each Linear Equation by finding the Intercepts. Carefully mark and number your axes!!

a) $2x+6y=180$



b) $10x+5y=210$



4) Put each line into Slope-Intercept form

a) $8x+6y=96$ b) $4x+18y=90$

SECTION 1.2: SLOPE OF A LINE AND CREATING LINEAR EQUATIONS

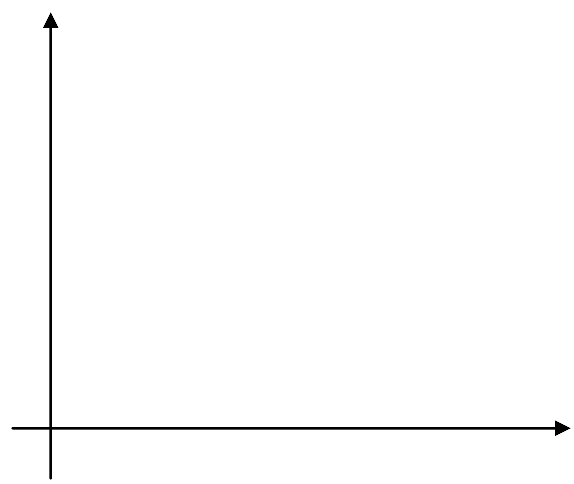
Most of the Equations you will use in this course will be in the General Form, $cx + dy = e$. The Slope of the Line is not visible in this form, but can be quite easily be found by converting the equation into Slope-Intercept form, $y = mx + b$, in a simple 2-step process: (i) "move" the term with *x* to the other side, and (ii) divide both sides of the equation by *d*. Do so on the following examples:

a) $4x + 3y = 24$ b) $2x + 6y = 60$ c) $8x + 20y = 600$

In each of the examples, you should now have an equation in $y = mx + b$ form, where *m*, the coefficient of *x*, is your Slope: (a) -4/3, (b) -1/3, (c) -2/5

The Slope is a very important value of any Line, offering us valuable information in certain contexts and usages of the Line. But, is it useful for graphing the typical Line we will see in this course? Let us graph the line in letter (c) above two ways.

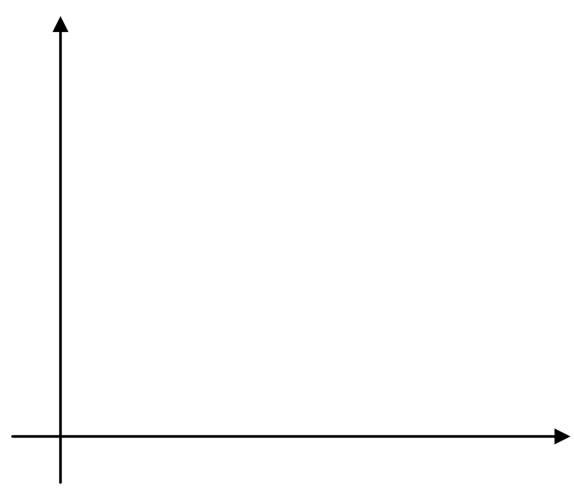
i) Using Slope. You should have the equation $y = \frac{-2}{5}x + 30$. So, the 30 indicates a y-intercept at (0, 30). Plot this and then use the slope to go down 2 and right 5 to plot a second point. Draw the line through these 2 points.



ii) Starting with $8x + 20y = 600$, find both intercepts like we did in the 1.1 worksheet. Plug in 0 for *y* to get the *x-intercept* and then plug in 0 for *x* to get the *y-intercept*.

How will you number your axes? Try using 10's

Does the line in (i) hit the X-Axis at the same point as your *x-intercept* in (ii)? Did it even hit the X-Axis?



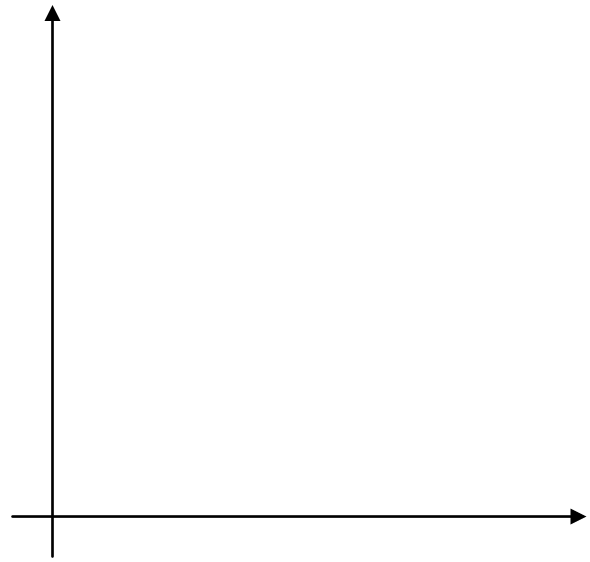
The previous example is meant to show that when we can find BOTH intercepts, it is a superior choice to graphing the typical line we will encounter in this course. However, the lines we have been graphing in this way, in $cx + dy = e$ form have all had $e \neq 0$. What about when $e = 0$? There will be only one Intercept, which is at (0, 0). So, how should we approach graphing these lines? Again, slope can be used but might not be the best choice when "larger" values are in play. Let us graph $2x - 3y = 0$, but suppose we also are graphing the Line $4x + 5y = 600$.

The Line $4x + 5y = 600$ has as its Intercepts $(150, 0)$ *and* $(0,120)$, which you should verify for yourself. How should we number our Axes for these Intercepts? By 1's? By 10's? Maybe by 30's? Use 30's.

Draw the line through $(150, 0)$ *and* $(0,120)$

Now take $2x - 3y = 0$, what is the Slope? Do you get $m = 2/3$? Try using the Slope and (0, 0), moving Up 2 and Right 3. Does this work very well? Probably not.

So, what if we decide to use $x = 60$? What is *y*? You should get *y* = 40, so the point (60,40). Plot (60, 40) and decide whether your line using "Up 2 and Right 3" was accurate or not.

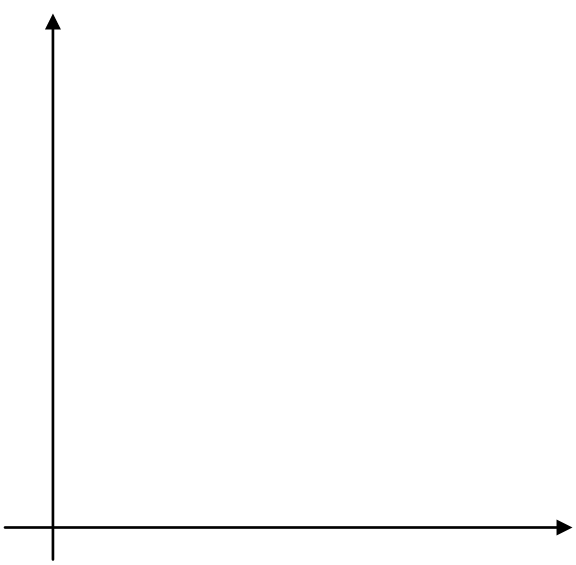


Try this on the following pairs of lines, numbering the Axes as suggested in each.

$2x - y = 0$

$5x + 10y = 500$

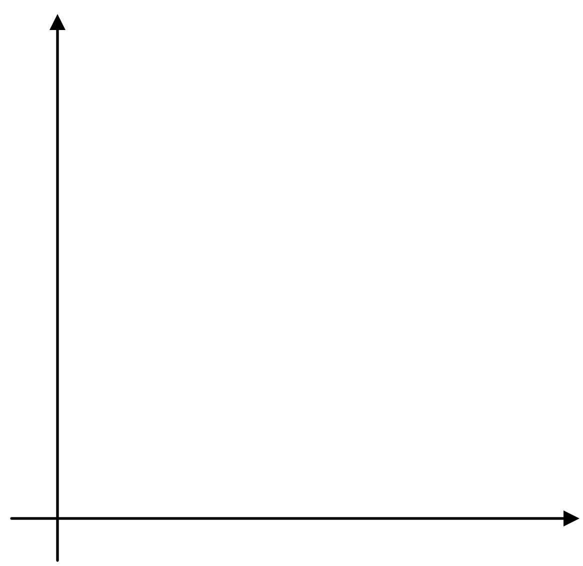
By 25's



$x - 3y = 0$

$12x + 18y = 540$

By 10's



The Slope of a Line is relatively simple to determine when we already have an equation for the Line, but what about when we do not? For this situation, we go back to an Algebra formula:

$Slope = m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where the *x's* and *y's* are from two separate coordinate pairs.

Practice use of the formula on each of the below sets of coordinates(leave answers in FRACTIONS):

a) $(4,10)\ and\ (8,20)$ b) $(0,16)\ and\ (8,12)$ c) $(30,125)\ and\ (45,210)$

Now, let us build an equation for a Line when all we have to start are two coordinate pairs. Again, we go back to an Algebra formula: $y - y_1 = m(x - x_1)$. Recall that *m* is our slope, determined by the formula above, and that the $x_1\ and\ y_1$ come from EITHER coordinate pair. Try this on each of the previous examples, using the Slopes you determined above.

a) $(4,10)\ and\ (8,20)$ b) $(0,16)\ and\ (8,12)$ c) $(30,125)\ and\ (45,210)$

APPLICATION: Suppose a company determines that it costs \$3200 to build 40 items, and it costs \$4950 to build 75 items. They would like to have a Linear Cost Equation in Slope-Intercept form. What should be *x* and what should be *y*? Ask yourself this: "Is a cost of \$3200 the result of building 40 items, or is building 40 items the result of a cost of \$3200?" The first makes more sense, doesn't it? Use that idea to build the two coordinate pairs, find the Slope, and lastly build the full equation in $y = mx + b$ form.

Note: the value of *m* is now a "per item cost", while *b* is reflective of a "fixed cost".

SECTION:1.3 THE INTERSECTION POINT OF TWO LINES

The Coordinates of a point of Intersection of two lines is often of great importance to us, so we want to use proper methods to determine these Coordinates, as opposed to "guessing" by just looking at a graph. The textbook and MyMathLab "help" will suggest one method, which we will use below, but then compare it to the method you should have learned in Algebra.

Find the Intersection of the two Lines: $\begin{matrix} 2x+5y=16 \\ 7x+3y=27 \end{matrix}$

Method I(text/MML): solve each Equation for *y*, into Slope-Intercept form. Once you have done so, use the Transitive Property to set the "*x* sides" equal to one another. Solve this equation for *x*. Once you have *x*, plus it back into an original equation to solve for *y*.

Did you get $\frac{-2}{5}x+\frac{16}{5}=\frac{-7}{3}x+9$? How fun was it to solve this fraction-filled equation?

Method II(from Algebra): Choose either Variable, *x* or *y*, and multiply each equation, on both sides, by the Coefficient from the other equation, introducing a Minus Sign to one of them, if necessary, so that you will now have +/- the same coefficient on that chosen variable. Let us select *x*, which means we multiply the first equation by 7, and the second by -2(to address the need for a Minus Sign). You should have a +14*x* and a -14*x* in your rewritten equations. Add those equations together, eliminating *x*. Then solve for *y*, and then use y plugged back into an original equation to determine *x*.

Were there any fractions to be handled in this process? Which of the 2 methods do you prefer?

Try each method again on the following exercises. Either will be allowed on Quizzes and Exams, this worksheet is simply trying to help you decide which works best for yourself.

a) $\begin{array}{l} 4x+5y=13 \\ x+3y=5 \end{array}$ b) $\begin{array}{l} 2x-3y=6 \\ 5x+y=32 \end{array}$ c) $\begin{array}{l} 12x+15y=195 \\ 20x+10y=250 \end{array}$ d) $\begin{array}{l} 8x+9y=26 \\ 12x+3y=32 \end{array}$

Application: Suppose two salespeople are newly hired by a company. The first asks for a weekly base salary of $225 plus a commission of $35 for each sale made, while the second one asks for a weekly base of $150 plus a commission of $40 per sale. (i) Build linear Equations to represent each salesperson's weekly earnings for sales of *x* items, and (ii) Find the number of sales that would see them with identical earnings for a week, plus the total weekly earnings for each in that case.

Application: A family goes to a ballgame and buys 4 sandwiches and 5 sodas for a total of $43.30, while another family purchased 6 sandwiches and 4 sodas for a total of $52.70. Build two equations using X = Price of a Sandwich and Y = Price of a Soda, and then solve for these two prices.

1.3 EXERCISES

1) Use Elimination to find the intersection of each pair of lines

a) $\begin{aligned} x+5y&=14 \\ 7x+3y&=34 \end{aligned}$

b) $\begin{aligned} 6x+5y&=35 \\ 4x+3y&=22 \end{aligned}$

c) $\begin{aligned} 10x+15y&=170 \\ 25x+20y&=320 \end{aligned}$

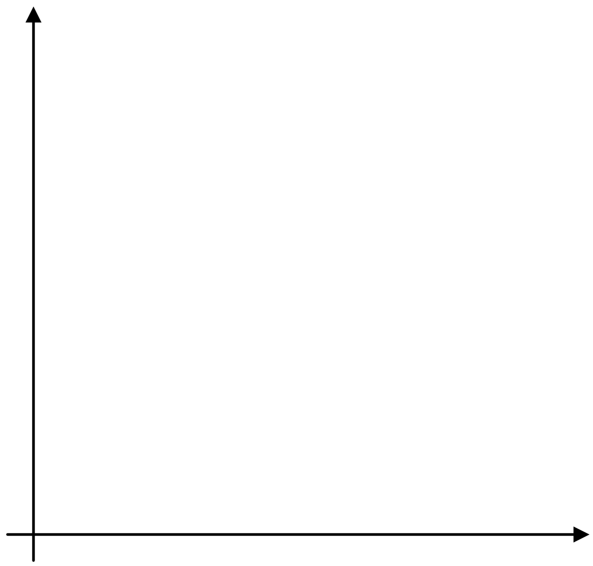
d) $\begin{aligned} 40x+20y&=440 \\ 30x+75y&=1050 \end{aligned}$

2) A furniture factory makes couches and chairs. Each couch needs 6 hours of construction time and 4 hours of upholstering time, while each chair needs 2 hours of construction and 3 hours of upholstering. Each day, the factory has available a total of 90 hours for construction and 85 hours for upholstering. Build an equation to represent construction hours, and a second for upholstering, and then determine the intersection of the two lines, which gives the number of couches and chairs built where we use up all of the available hours daily.

SECTION 1.4: LINEAR REGRESSION

Linear Regression is just one of MANY regression models used in statistical modelling of real data for the purpose of making predictions for unknown values based on known values. We will look at finding the Linear Regression Line both by use of formulas and also by use of our graphing calculators.

First, a simple example to work through, with the following x,y pairs: $(1,3),(3,4),(4,7),(7,8)$. Mark both axes by 1's and plot the four coordinate points below. Try to be very consistent with your tick-marks.



X	Y	X^2	X*Y
1	3		
3	4		
4	7		
7	8		

Now, try to draw a line through the plotted points, what is called a "Line of Best Fit" because it should be as close as possible to all of the points, and will not necessarily hit any of them exactly. Once we have an equation for the mathematically determined line, you can see how well you did.

Next, fill in the table above, where the X^2 column is the Square of the respective X-coordinate, and the X*Y column is the product of each X,Y pair. Then total up each column, in the bottom of each column.

The total in Column X is described in the formulas below by Σx, and similar for the other column totals. Use the formulas to first determine the Slope value, *m*, and then use *m* in the formula for *b*, the Y-Intercept.

$$m = \frac{N \cdot \Sigma x \cdot y - \Sigma x \cdot \Sigma y}{N \cdot \Sigma x^2 - (\Sigma x)^2} \qquad b = \frac{\Sigma y - m \cdot \Sigma x}{N}$$

where N = # of X,Y pairs

Did you get this? $y = 0.88x + 2.2$ Does your sketched line seem to agree?

Use the formulas $m = \dfrac{N \cdot \Sigma x \cdot y - \Sigma x \cdot \Sigma y}{N \cdot \Sigma x^2 - (\Sigma x)^2}$ *and* $b = \dfrac{\Sigma y - m \cdot \Sigma x}{N}$ on the below sets of values, rounding each of *m* and *b* to 2 decimal places, as necessary. BUT, keep 5 places(or the fraction) for *m* when plugging into the formula for *b*. The line for each is under the respective tables. Did you get these?

X	Y	X^2	X*Y
4	2		
7	4		
9	8		
12	11		
13	13		

$y = 1.24x - 3.57$

X	Y	X^2	X*Y
21.3	442		
33.7	576		
38.6	612		
44.5	729		
48.3	745		
53.8	854		

$y = 12.47x + 160.45$

Starts to get pretty unwieldy with more X,Y pairs and/or bigger numbers in the mix, doesn't it?
Some of the burden of handling these calculations can be placed onto our calculators. The recommended calculators are the TI-83 or TI-84, and so the instructions will be based on using those models. As said at the start of the semester, those choosing to use a TI-89 or nSpire or Casio models will need to locate the proper functionality on their machines.

1) Hit the "STAT" key and select the first option available, "EDIT". Enter the values of X into L1 and the values of Y into L2. To exit the EDIT screen, hit "2nd" and then "MODE"(so QUIT, above MODE).
2) Hit "STAT" again, arrow to the "CALC" column and select 2-Var Stats. The default usage of this function is lists 1 and 2, so you need not tell the calculator which lists. However, if you do use other lists than 1 and 2, you can place those list names after 2-Var Stats by hitting "2nd" and then "3" for L3, as an example. And use a Comma, the button directly above 7, to separate you list choices.

In 2-Var Stats, you should see the values for N, Σx , Σy , Σx^2 , *and* Σxy , all of the necessary values for use in the two formulas we have been using so far. Verify these values for each of the examples at the top of the page, or use this to see what went wrong if you did not get the correct lines in those examples.

Our calculators have an even more powerful tool, and computer software will have even more advanced usages than this. Those of you who will some day need to use these sorts of calculations and modelling techniques will surely do so on those advanced software packages, as opposed to filling out tables and using formulas like we did above. So, now we look at what the TI's offer us in this way.

Take the previous examples X,Y sets and again place them into the STAT EDIT screen as lists L1 and L2. Remember to use 2nd/MODE(Quit) to exit the screen. Then, once again, hit STAT and arrow over to CALC. Now arrow down to LinReg(ax+b) and select this function. Just like with 2-Var Stats, the default usage is for L1 and L2, but can be used for any lists as before by placing their names after the basic command.

Verify that you get the same lines as we found by using the formulas for each of the above examples.

Now, we use the lines to make predictions.

i) Take the line $y = 1.24x - 3.57$ and suppose we wish to explore having an "input" value of $x = 10$(which notice was not in the table we used to find the line). What does the line predict for y?

On the other hand, what if we have a goal for y, say $y = 16$. What x would be necessary according to our line?

ii) now use the line from the second table, $y = 12.47x + 160.45$

a) If a target of 700 in output is desired, what input is needed?

b) An input of 51.2 has been discovered, what would the line predict for an output?