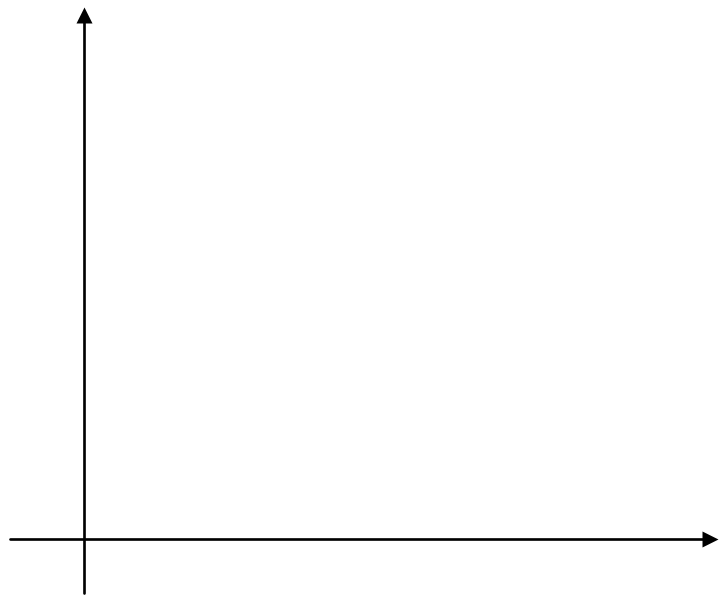
SECTION 3.1: GRAPHING LINEAR INEQUALITIES AND FEASIBLE REGIONS

We start with a reminder of the smart way to graph a Linear Equation for the typical example we see in this course, namely using BOTH X- and Y-Intercepts, when available.

EXAMPLE: Graph the Line $8x+6y=240$

a) Determine the Coordinates of BOTH intercepts
b) Choose reasonable values to mark the X- and Y-Axes
c) Plot and Label each intercept with their coordinates
d) Sketch the Line



Now, suppose instead of just a Linear Equation, we have a Linear Inequality: $8x+6y\le 240$
In Algebra coursework, you should have been taught to shade one side or the other, where you would choose the "good side", as this represented the half of the plane satisfying the Inequality.

However, in Math 125, we need to shade the "bad side" of our graphed line, so that when we do this on multiple inequalities, we will leave blank an area that is still "good" for every one of those inequalities.

Which side of the above graphed line should you shade? To decide, select a Coordinate Point that is NOT on the line itself. Try both $(0,0)$ *and* $(30,30)$ by plugging them into $8x+6y\le 240$. Which one leaves you with a true Inequality statement? Shade the side opposite the "good point", which is then shading the side with the "bad point".

On the SAME axes above, now graph using all of the same steps: $10x+20y\ge 200$

HINT: quite often, $(0,0)$ is available as a "test point" and as a great choice when this is so

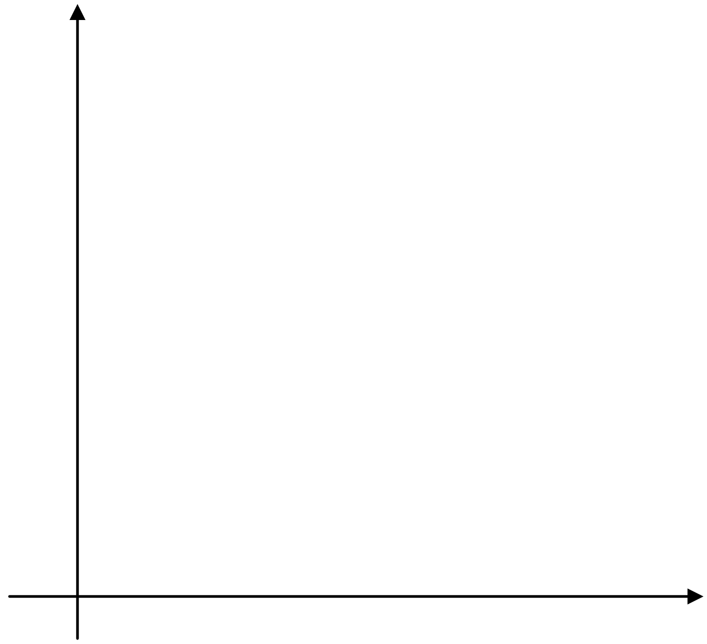
FEASIBLE REGIONS: Also called Feasible Sets(and thus abbreviated as FS) are the region on a graph where multiple Inequalities "agree", meaning that all of them can be satisfied by every coordinate pair this FS represents, both its boundary line segments(or half-lines) as well as all points in its interior.

GRAPH: place all three on the same axes

$5x + 6y \le 300$

$x \ge 18$

$y \ge 10$



Once all 3 have been shaded, do you have an unshaded triangle? That is the FS for these Inequalities.

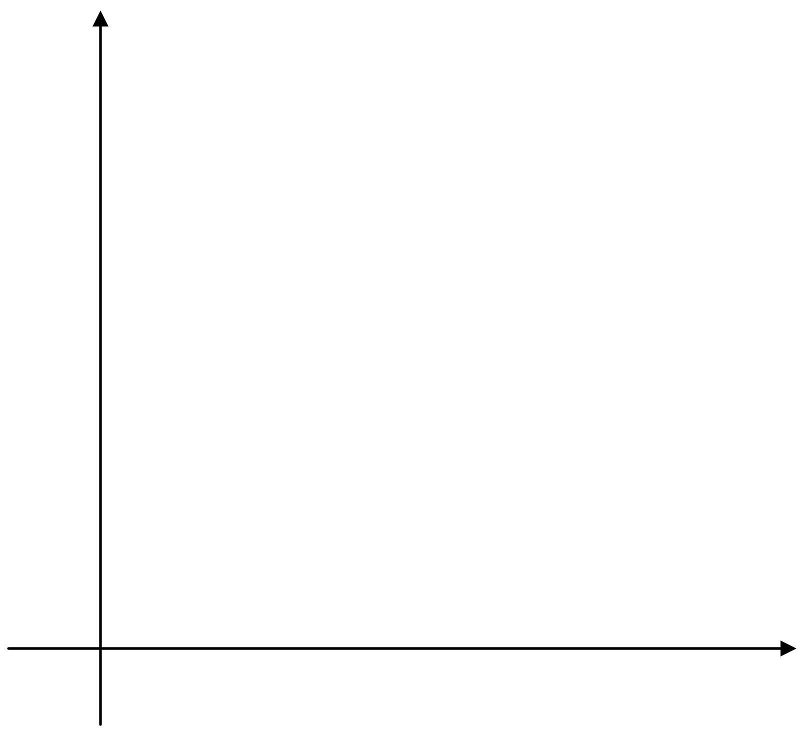
Now that we have a well-defined FS, determine the Coordinates of each of the 3 "corners" of the FS.

EXERCISE: Graph the below Inequalities on a single pair of axes, and then determine the coordinates of each corner of the FS once it is completed.

$2x + 6y \le 90$

$5x + 3y \le 105$

$x \ge 0$, $y \ge 0$

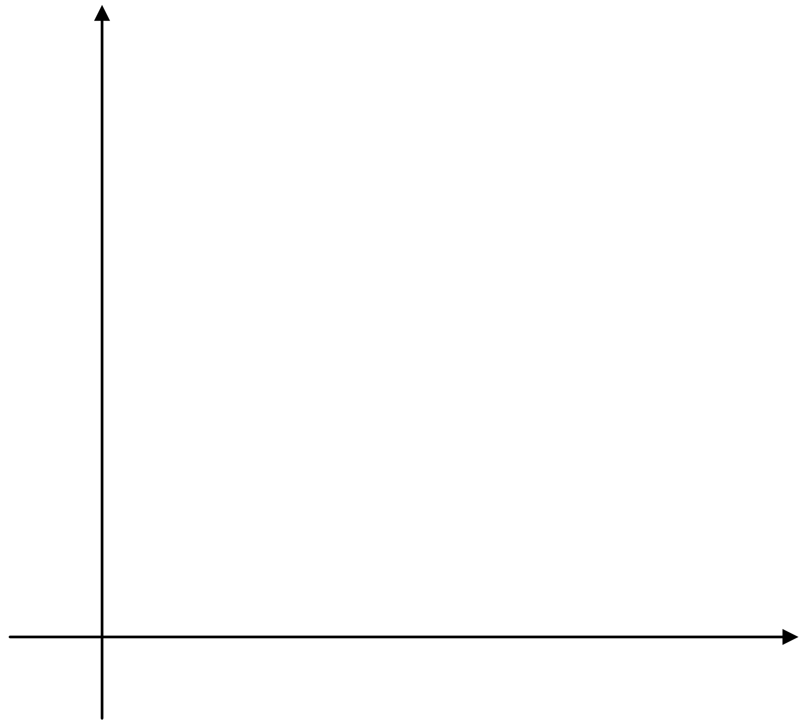


EXERCISE: Graph the below Inequalities on a single pair of axes, and then determine the coordinates of each corner of the FS once it is completed.

$x + 4y \le 40$

$3x + 4y \le 120$

$x \ge 12$

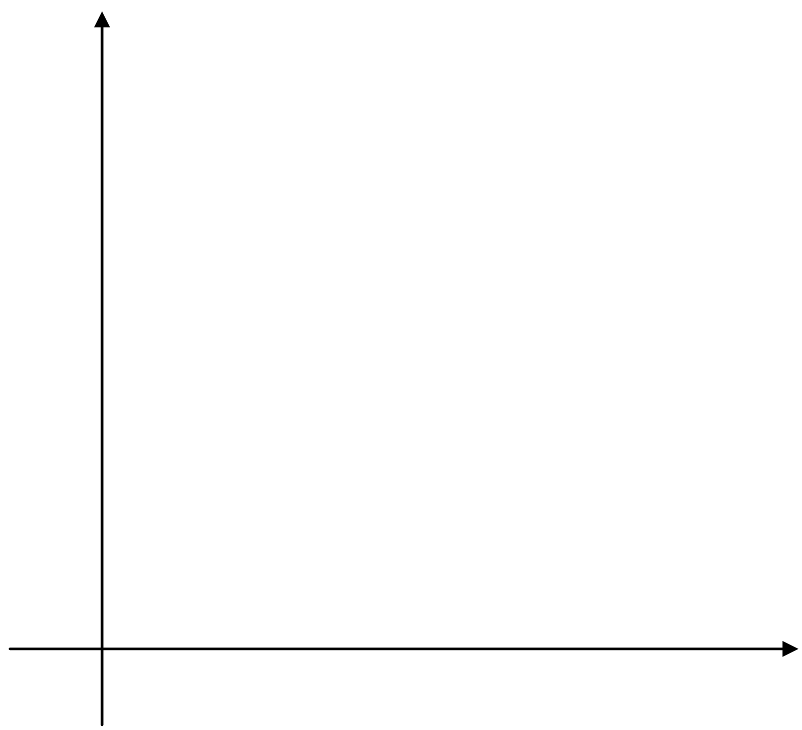


EXERCISE: Graph the below Inequalities on a single pair of axes, and then determine the coordinates of each corner of the FS once it is completed.

$3x + 2y \ge 48$

$3x + 4y \ge 72$

$x \ge 0$, $y \ge 0$



SECTION 3.2: THE LINEAR PROGRAMMING PROBLEM

As described in lecture, a *Linear Programming Problem* consists of three main components: an Objective Function, a set of Constraints, and Non-Negativity Constraints. This section is mainly for the purpose of converting applied situations(word problems!!) into properly constructed Linear Programming Problems(LP problems from now on). Sometimes, but NOT always, a table setup provides us a useful structure for laying out the information that easily translates into the constraints and objective function. We begin with such examples.

EXAMPLE: WidgetWorld builds two products, a basic widget and a deluxe widget. Each basic widget needs 2 hours of assembly and 1 hour of painting, and each deluxe widget needs 3 hours of assembly and 4 hours of painting. The workroom has available each day 120 hours for assembly and 120 hours for painting. Profits are $12 for each basic model and $15 for each deluxe model. Construct a full Linear Programming Problem to help WidgetWorld earn the greatest profits on a daily basis.

i) **Name your variables and what they refer to**. Identify what quantities in the application are unknown and will specifically be determined. Here it is how many basic widgets and how many deluxe widgets. So:

x = # ___________________ and y = #___________________

Use the topics described by our variables as the headings for the columns in the table below, and then list the "inputs" that go into these items as the row headings. Then fill in the table with appropriate values. Often, we put the "objective" in the bottom row, here that is Profits.

Inputs ∨	X=	Y=	Available
Profit			

ii) **Build a proper Objective Function**. Often, the aspects of this are given at the end of the application write-up, yet will be the first line of the LP Problem. **Always** include whether we are wanting to *Minimize* or *Maximize* in achieving our goal.

iii) **Construct all necessary Constraints**, including the **Non-Negativity Constraints**. In the application wording will be something to suggest whether we need go only up to, but not over, some value(perhaps the word "available") or we need to reach at least some, or go above(perhaps a "required" amount). This will help you decide which to use: $\leq$ *or* $\geq$

EXERCISE: Construct the LP-Problem for the following situation.

A national organization wants to increase their membership and is hoping to catch the attention of younger adults. When they do a college campus visit, it costs $100, takes 4 person-hours, and generates 32 new members. When they visit a shopping mall, it costs $40, takes 3 person-hours, and generates 18 new members. For this year's membership drive, they have budgeted $8000 and have 390 person-hours available. How many campus visits and mall visits should the organization make to obtain the greatest amount of new members, and how many new members will they enroll?

i) **Name your variables and what they refer to**. Then fill in the table below

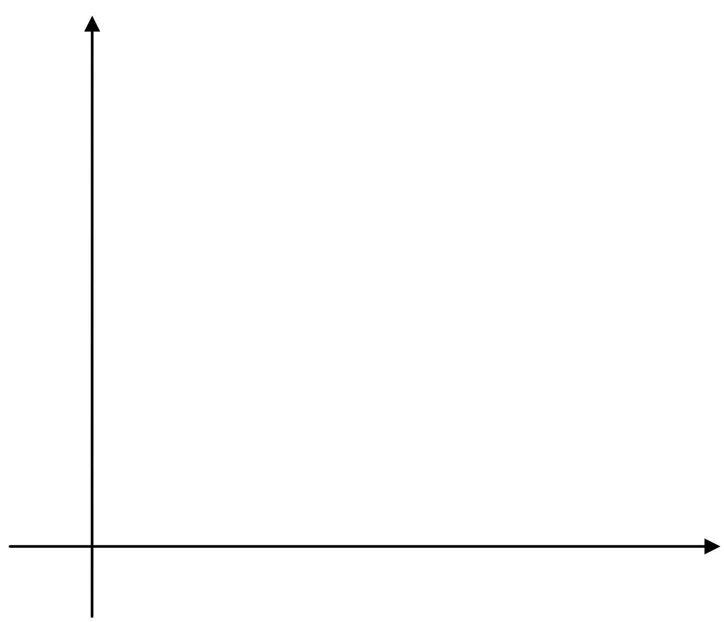
x = # ____________________ and y = #____________________

Inputs ∨	X=	Y=	Available

ii) **Build a proper Objective Function**.

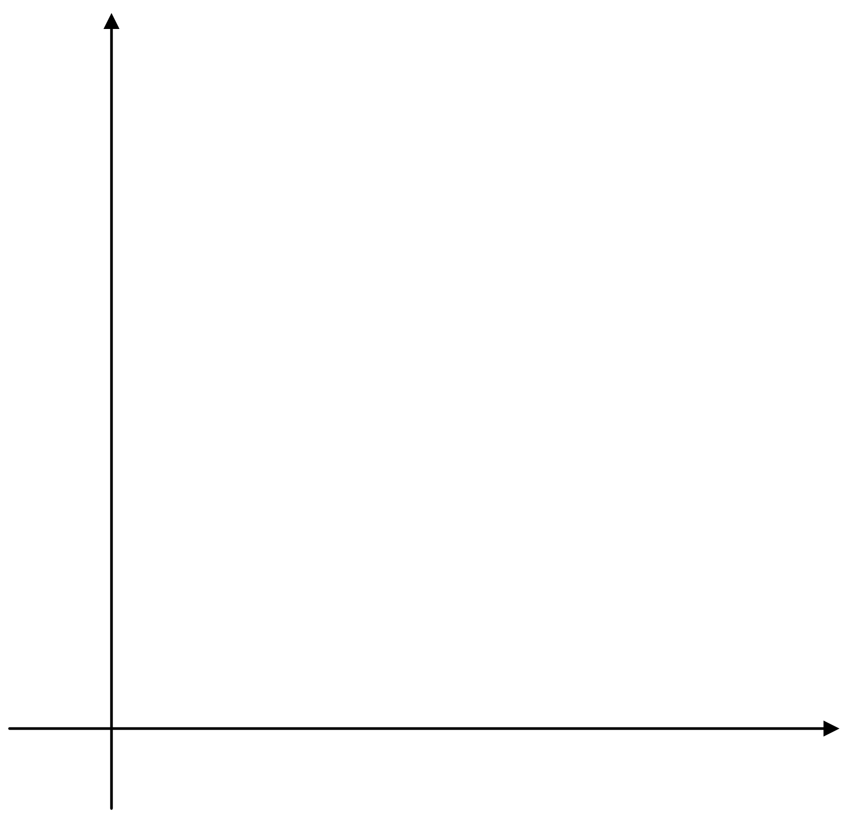
iii) **Construct all necessary Constraints**, including the **Non-Negativity Constraints**.

iv) Graph the Feasible Set and determine the coordinates of ALL corner points.



EXERCISE: EnergeeBar is designing a new breakfast bar and plans to use two natural ingredients in each bar. Ingredient A has 3g of protein per ounce, 4g of fat per ounce, 6 units of vitamin B_{12}, and costs \$0.12 per ounce, while Ingredient B has 2g of protein per ounce, 6g of fat per ounce, 2 units of vitamin B_{12}, and costs \$0.08 per ounce. The nutritionist wants at least 60g of protein, 60mg of fat, and 84 units of B_{12} per bar. How many ounces of each ingredient should be used to create a breakfast bar meeting the nutritional requirements at least cost?

Build a properly constructed LP-Problem. Then graph the problem and determine the coordinates of all corner points of the Feasible Set. Use of a table is not required, but is still recommended.



EXERCISE: ScrappleLot company owns two scrap metal plants. Plant I produces 3 tons of iron, 1 ton of copper, 2 tons of aluminum daily, at a cost of $600, while Plant II produces 2 tons of iron, 2 tons of copper, 1 ton of aluminum daily, at a cost of $800. The general manager wants Plant II to be run at least twice as many days as Plant I in filling an order received for 24 tons of iron, 20 tons of copper, and 12 tons of aluminum. How many days should ScrappleLot run each plant fulfill the order at least cost, and what is the least cost?

Build a full LP-Problem. A table is not required, but is still recommended. DO NOT GRAPH.

QUESTION: Do you have an Inequality constraint for the line that says: "The general manager wants Plant II to be run at least twice as many days as Plant I"? Did a table help to create this? Most likely, the table did not help, this being a perfect example of why tables are helpful only sometimes.

To construct a proper constraint for this line, start by setting up the basic comparison, Plant II to Plant I:

Plant II = y "versus" Plant I = x

Which should be "multiplied by 2"? If you are not sure, try a specific number for Plant I, say 10. If Plant I is to be run 10 days, what does the line suggest is necessary for plant II? If at least twice as many, then it must be run 20 or more, correct? Does that not mean we need to multiply Plant I, so x, by 2? Therefore, we now have:

$$y \qquad 2x$$

Now, since we want Plant II "at least" that much, we have this: $y \geq 2x$. Other acceptable forms of this same constraint would be these: $-2x + y \geq 0 \quad or \quad 2x - y \leq 0$

SECTION 3.3: SOLVING THE LP-PROBLEMS BY USE OF THE FUNDAMENTAL THEOREM

The Fundamental Theorem of Linear Programming tells us that a Convex, Linear-based Feasible Set will realize an Optimal value to a Linear Objective Function, either at a corner of the Feasible Set or at adjacent corners along with the line segment connecting those adjacent corners. Thus, our graphing solution process to LP-Problems can be summarized as follows:

i) Construct the full LP-Problem from the application/word problem
ii) Sketch the graph of all constraints, shading properly to determine the Feasible Set
iii) Determine the coordinates of ALL corners of the Feasible Set. List them.
iv) Plug each of the corners into the Objective Function, and identify the desired Maximum or Minimum value, as stated in the Objective Function. The values of *x* and *y* are also useful in applications.

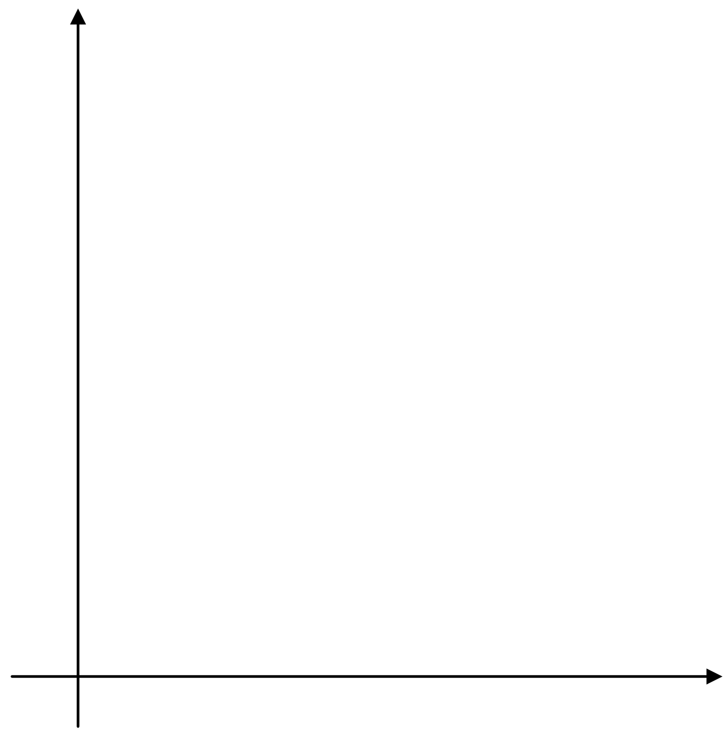
EXAMPLE: The below LP-Problem should be what you constructed for the WidgetWorld example at the start of the 3.2 worksheet. Follow the steps above to find the Maximum Profit, and also determine how many of each type of widget, basic and deluxe, should be made to realize this level of profit.

$Maximize:\ 12x+15y \quad s.t.$

$2x+3y\le 120$

$x+4y\le 120$

$x\ge 0\ ,\ y\ge 0$



Max Profit: ___________ , # Basic Widgets = _________ , # Deluxe Widgets = __________

EXERCISE: For a fully *bounded* Feasible Set, the Fundamental Theorem says we can find BOTH a Minimum and a Maximum for a given Objective Function. Graph the below set of constraints, and obtain the list of ALL corners of the Feasible Set.

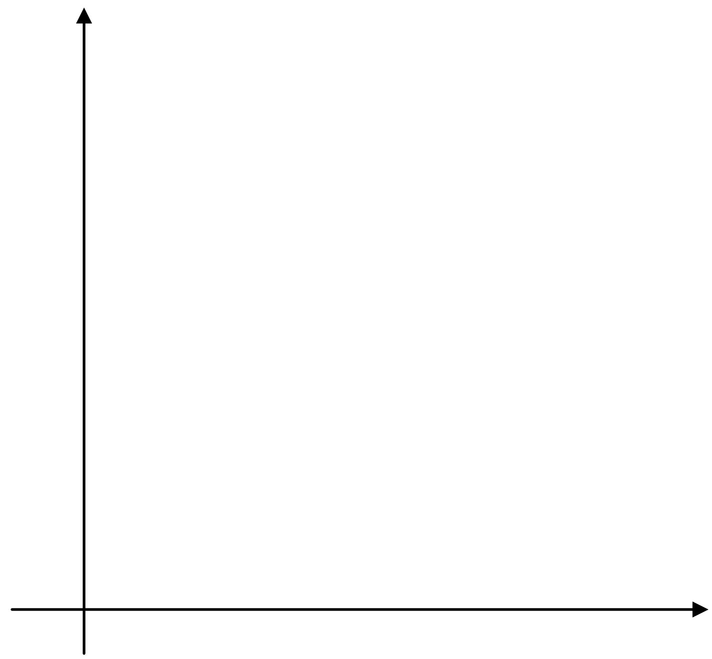
$x + y \leq 800$

$x + y \geq 250$

$x \leq 600$

$y \leq 500$

$x \geq 0$, $y \geq 0$



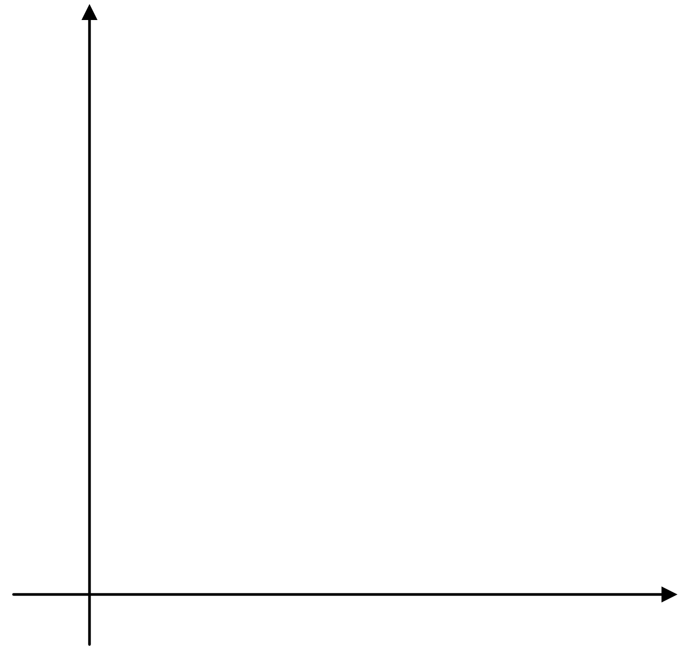
Now suppose we have the Objective Function: $5000 - 12x + 15y$. Plus all of the corners into the function and find BOTH the minimum and maximum values.

What if instead, the Objective Function is this: $5000 + 18x - 14y$? You should get both a maximum and minimum again, but they are different values, and at different corners.

One Feasible Set will provide different solutions, all dependent on the Objective Function.

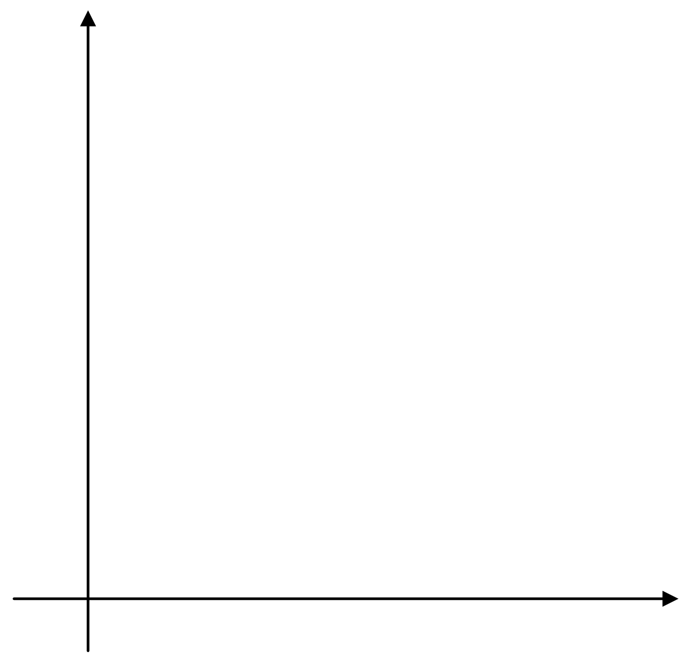
EXERCISE: Graph the Feasible Set, determine all corners, and find the minimum

$Minimize: 18x+30y \ \ s.t.$
$3x+2y \geq 24$
$5x+4y \geq 46$
$4x+9y \geq 60$
$x \geq 0 \ , \ y \geq 0$



EXERCISE: Graph the Feasible Set, determine all corners, and find the maximum

$Maximize: 18x+9y \ \ s.t.$
$x+5y \leq 60$
$x+y \leq 16$
$2x+y \leq 26$
$x \geq 0 \ , \ y \geq 0$

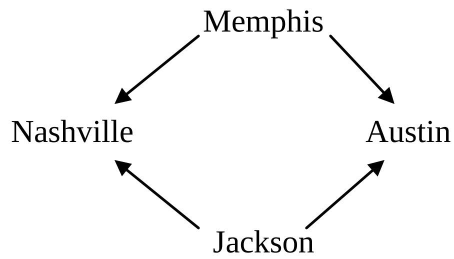


SECTION 3.4: SOLVING LP-PROBLEMS WITH 3 OR 4 UNKNOWNS USING 2 VARIABLES

One of the classic uses for LP-Problems is in solving shipping problems, where in this course we take a look at a fairly simple setup which allows us to graph a 2-variable Feasible Set. It is recommended the student uses the below described process for configuring the information in such way as to assist in building the correct set of constraints.

Git-tar Corp. makes guitars at two workshops, Memphis, with 90 guitars in stock, and Jackson, with 120 in stock. They have retail stores on Nashville, which needs 60, and Austin, which needs 75. It costs $15 to ship each guitar from Memphis to Nashville, $20 from Memphis to Austin, $24 from Jackson to Nashville, and $18 from Jackson to Austin. How many should be shipped in each of the four routes, and at what minimum total cost?

The diagram at the right indicates the direction of movement.



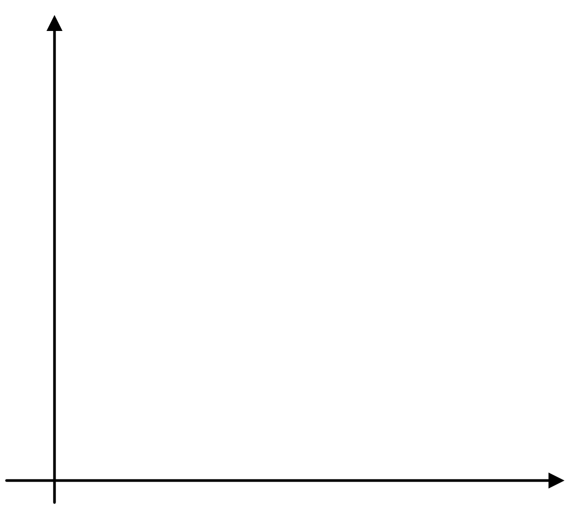
It is best to choose the two quantities being shipped out of a city as *X* and *Y*. Do so for Memphis. Now decide how to represent the quantities shipped out of Jackson based on how many are needed in Nashville and Austin. You **MUST USE** *X* and *Y* to do so.

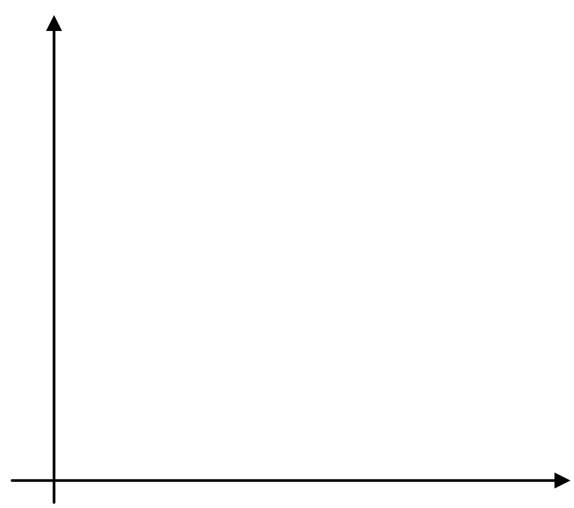
List each quantity next to the city, along with whether it “has” or “needs” that quantity. The clues to setting up constraints lie in these thoughts: if a workshop only has so many on hand, it cannot ship out more than that amount, and each quantity shipped(in the 4 routes) cannot be negative. Simplify, as needed, to put each constraint into preferred form.

Objective Function: to build this, list the shipping cost next to each of the four routes, multiply that cost times each respectively described quantity shipped, and add them all up into one single function. Simplify as much as possible.

EXERCISE: Redo the previous example, but this time select Jackson as the origination point, label the quantities leaving Jackson as *X* and *Y* on a new diamond-shaped diagram. Fill out the rest of the diagram accordingly and then build all necessary constraints, plus a new Objective Function.

You should now have two separate, similar but different, LP-Problems. To see that both actually give the same solution, graph both below, and then detail each of the four routes' quantities while also noticing you should have the same minimum cost.





EXERCISE: A produce company regularly ships apples, bananas, and cherries to Chicago. The truck carries exactly 180 crates and is always full on each trip. Their customers want them ship at least 20 of each type of fruit. The sales manager wants at least as many crates of cherries as apples, and at least twice as many bananas as cherries. The profit is $15 on each crate of apples, $10 on each crate of bananas, and $24 on each crate of cherries. How many crates of each should be sent each shipment to realize the most profit, and what will be that amount of profit?

We have 3 unknowns, in how many crates of each of three types of fruit. In order to model this using only 2 variables, we recall that all three cases of fruit quantities will ALWAYS total 180 crates. So, for simplicity, we will use X = # crates of apples and Y = # crates of bananas. Now, how should we model the # crates of cherries?