SECTION 2.5: PROPERTIES OF DERIVATIVES

The most frequently used property for derivatives is the Power Rule: $\dfrac{d[x^n]}{dx} = n \cdot x^{n-1}$

Use this property to find the derivative of each of the following functions:

a) $f(x) = x^4$ b) $g(x) = x^{10}$ c) $Q(x) = x$ d) $F(x) = x^{7/4}$

What if a function is not in x^n form? Perhaps we can rewrite the function using exponent properties from our Algebra toolkit. As a reminder, these properties are often useful:

$\dfrac{1}{x^n} = x^{-n}$ $\sqrt{x} = x^{1/2}$ $\sqrt[n]{x} = x^{1/n}$ $\sqrt[n]{x^m} = x^{m/n}$

Use the properties, as needed, to rewrite each of the functions below, THEN use Power Rule

a) $f(x) = \dfrac{1}{x^6}$ b) $g(x) = \sqrt{x}$ c) $Q(x) = \sqrt[4]{x}$ d) $F(x) = \dfrac{1}{\sqrt{x^3}}$

Next, mix in the Derivative property: $\dfrac{d[k \cdot f(x)]}{dx} = k \cdot \dfrac{d[f(x)]}{dx}$

a) $f(x) = 5x^3$ b) $g(x) = 8\sqrt{x}$ c) $Q(x) = \dfrac{2}{3}x^6$ d) $F(x) = \dfrac{5}{x^2}$

One more property to incorporate into the mix: $\frac{d[f(x)\pm g(x)]}{dx}=\frac{d[f(x)]}{dx}\pm\frac{d[g(x)]}{dx}$, which is telling us we can simply take the Derivative one function, or term, at a time. Use this with the previously explored properties to find derivatives of each of the following:

a) $f(x)=x^7+4x^3-8x^2$

b) $y=x^5-7x-8$

c) $g(x)=6\sqrt{x}+12\sqrt[3]{x}$

d) $h(x)=\frac{4}{x}-\frac{3}{x^6}$

Sometimes functions need some simplifying or rearranging first, using Algebra. What can you do to each of the following using Algebra to make them easier to apply Derivative properties? Find derivatives of:

a) $f(x)=(4x-3)(x^2+5)$

b) $g(x)=2(x-4)^2$

c) $h(x)=\frac{x^6+4x^3-8x}{x^3}$

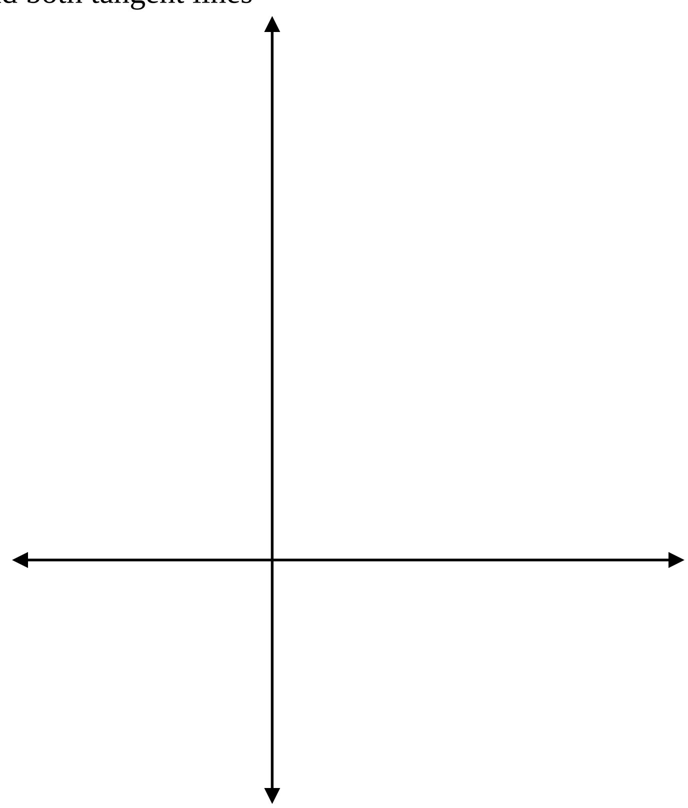
Now that we have established quicker methods of finding derivatives than use of the Definition of the Derivative, let us begin using the Derivatives for their most basic usage. $f'(c)$ gives us the Slope of a Line Tangent to f(x) at $x = c$. That Slope then allows to build a full Equation of that Tangent Line.

Given $f(x) = 6x - x^2$, first verify that the point $(2,8)$ is indeed on the graph of f(x).

Next, find the derivative, $f'(x)$, and then use it to find the tangent slope at $x = 2$.

We now have two essential pieces of information needed to construct an Equation of a Line, the Slope and a Point. Use the Algebra based Point-Slope formula: $y - y_1 = m(x - x_1)$

Now, using the same $f(x)$ *and* $f'(x)$, find the Equation of the Line tangent to f(x) at $x = -1$. Then sketch all three items on the same set of axes: $f(x)$ and both tangent lines



SECTION 2.6: DIFFERENTIALS AND LINEAR APPROXIMATIONS

In lecture, as well as in the textbook, we developed a formula to approximate the change in output to a function, specifically the following:

$dy = f'(x_0)\Delta x$ where x_0 is an "initial point", where we place a Tangent Line and Δx is the change in x

For example, suppose we would like to approximate the value of $\sqrt[3]{8.24}$. You should choose an appropriate function and a value for x_0 that is logical to place the Tangent Line. Once you have chosen the x_0 value, decide what value Δx has. Here, we select $f(x) = \sqrt[3]{x}$, $x_0 = 8$, *and* $\Delta x = 0.24$
First, find $f'(x)$ and then use it to calculate $dy = f'(x_0)\Delta x$ for this situation.

Remember this estimates the "Change in y", so we add this to $\sqrt[3]{8} = 2$ to get an approximation of $\sqrt[3]{8.24}$

Using the same setup of $f(x) = \sqrt[3]{x}$ *and* $x_0 = 8$, find an approximation for each:

i) $\sqrt[3]{8.6}$

ii) $\sqrt[3]{7.64}$

Let us focus a little more closely on *dy*, the approximate ***Change in y***. This can be a particularly useful value to determine in business related function analysis. Suppose we have a Profit function as given by $P(x) = 24000 + 800x - 0.05x^2$, and we are currently producing/selling 500 items. First, for a point of reference, calculate our current Profit, so $P(500)$ = ???

Now, suppose the production of 20 more items is possible, and we would like to know what might happen to our total Profit if we boost production by that amount. If we adjust the our model, $dy = f'(x_0)\Delta x$, to $dP = P'(x_0)\Delta x$, which fits our current Profit function, we can use it to approximate that change in total Profits. Determine $P'(x)$, and using $x_0 = 500$ *and* $\Delta x = 20$, calculate this approximate change.

An actual Change in *y* can be found by $\Delta y = f(x_0 + \Delta x) - f(x_0)$, so here $\Delta P = P(x_0 + \Delta x) - P(x_0)$ Calculate ΔP and see if our approximation *dP* seems reasonably close.

Remember, we have not before produced 520 items, so would not truly know P(520), and so ΔP is not something we can do "in the real world", while *dP* often is possible. But now that we have calculated *dP*, do you think we would desire making those extra 20 items?

Next, suppose we instead are at a production level of 1000 items, and want to know if 20 more is a good idea. It sure seemed to be at 500, but how about at 1000? Redo the same analysis above, but this time with $x_0 = 1000$. What do you get for *dP* ??

For a fuller understanding of what *dP* means, which is just an ***Approximate Change*** in Profit, not ***Total*** Profit, calculate P(1000) to see that we are making a Profit. But, it might go down if we produce more.

Exercise: Given cost, $C(x) = 225 + 10x$, and Revenue, $R(x) = 30x - 0.2x^2$:

a) Find the Profit function (Hint: Profits = Revenue – Costs)

b) Assuming a current production level of 40, estimate the changes in Revenue AND Profits if 5 more are to be produced

c) Assuming a current production level of 60, estimate the changes in Revenue AND Profits if 5 more are to be produced

Exercise: A popular downtown coffeeshop determines that the daily Price-Demand relationship for their signature LatteChino drink is $D = 1080 - 30p^2$ where $2 \le p \le 6$. Currently, the shop charges \$4.10 for the LatteChino. What are daily revenues on this drink?

a) Estimate the change in Demand if they change the price to \$4.25 per drink. What are daily revenues now?

b) Estimate the change in Demand if they change the price to \$3.90 per drink. What are daily revenues now?

SECTION 2.7: MARGINAL COST, REVENUE, AND PROFIT

As an opening skill-check, solve each Price-Demand function for price, *p*, where *x* is demand(or quantity)

i) $x = 1250 - 25p$ ii) $x = 2400 - 40p$ iii) $x = 2700 - 30p$

Now, using the first Price-Demand function in the form *p as a function of x*, build the Revenue function, where we use the basic notion that Revenue = Price*Quantity

R(x) = ____________________

Next, we find $R'(x)$, which is known at the Marginal Revenue function. We can use this in a similar manner to the concepts in Section 2.6 on Differentials, but what if we want to simplify that idea just a bit. In particular, if $\Delta x = 1$, then $dR = R'(x_0)\Delta x$ becomes a very simple $dR = R'(x_0)$.

We started with $x = 1250 - 25p$, solved for *p*, and then built R(x) and now have found $R'(x)$.

If we ask for the Change in Revenue from the 201st item, then $x_0 = 200$ and $\Delta x = 1$ means we simply find $R'(200) =$???

What is an approximate change in revenues from the 451st item?

What is an approximate change in revenues from the 751st item?

Does it make sense that the revenue generated from the 751st item is *negative*? Or is it more reasonable to think it is more the case that our total revenues on sales of all 751 items will fall slightly in comparison to if we had produced and sold 750 items?

Mostly, we are interested in whether Total Revenues are increasing or decreasing overall.

Exercise: Given Cost, $C(x) = 9000 + 20x$, and Price-Demand, $x = 1800 - 30p$, do the following:

a) Solve the Price-Demand equation for the price, p.

b) Build the Revenue function, R(x)

c) Find Marginal Revenue

d) Estimate the change in Revenues from the 501st and 801st items

e) Build the Profit function, P(x)

f) Find Marginal Profit

g) Estimate the change in Profits from the 501st and 801st items
Also calculate the total Profit on 500 items and 800 items

h) The Break-Even point is where there is no profit and no loss, so P(x) = 0. Find the Break-Even Points, and then also determine the Cost/Revenue values at each.

Average Cost and Marginal Average Cost: the average cost of x items is the total cost of those x items divided by how many items were built, which is of course, x. Therefore, we can build a function for Average Cost, denoted by $\overline{C}(x)$, which we call "C bar of x", as simply as $\overline{C}(x) = \frac{C(x)}{x}$. The Marginal Average Cost will, as you should expect, be the Derivative of $\overline{C}(x)$. We do not provide a formula for this because the construction of $\overline{C}(x)$ can vary greatly. Suppose $C(x) = 16000 + 120x - 0.25x^2$, $x \le 200$,

i) Build the Average Cost function, $\overline{C}(x)$ and simplify it as possible

ii) What is the average cost of 50 items? 100 items? 160 items? Round each to the nearest $0.01

iii) Find the Marginal Average Cost at each of 50, 100, and 160

Exercise: Given $C(x) = 2000 + 15x$,

a) Build the Average Cost function
b) Find the average cost of 20, 50, and 200 items
c) Determine the Marginal Average Cost
d) Find the marginal average cost of 20, 50, and 200 items