SECTION 4.1: FIRST DERIVATIVE AND RELATIVE MAXIMUM/MINIMUM

The mathematical concepts of how the First Derivative describes for us the slope of a function at any given point and how this helps us determine where that function is Increasing or Decreasing, as well as where the function has Relative Maxima or Minima, was shown in lecture. This worksheet is meant to give you practice in following a series of steps to gather up all of that information.

Find the Derivative of each function. Determine the X-values of all Critical Points(CPs for short), where either $f'(x)=0$ or where $f'(x)$ *Does Not Exist*. You must use various Derivative techniques among those learned in this course, as well as your Algebra skills, to do this properly.

i) $y=x^3+9x^2-4$

ii) $f(x)=x^3e^{-x}$

iii) $h(x)=6x^{5/3}-30x^{2/3}$

iv) $R(x)=\dfrac{4x}{x^2+1}$

IMPORTANT QUESTION: Do any of the above functions have a Vertical Asymptote? Always stop and ask this question after locating the Critical Points, because you need to add them to the mix in sign charts.

Find the Critical Points for $h(x)=\dfrac{x^2+16}{x}$

Does *h(x)* have any Vertical Asymptotes?
With this in mind, our "Partition Values" are $x=-4,0,4$, the full list of CPs and Vertical Asymptotes
We do not see much of this just yet, but will see it quite often in section 4.4

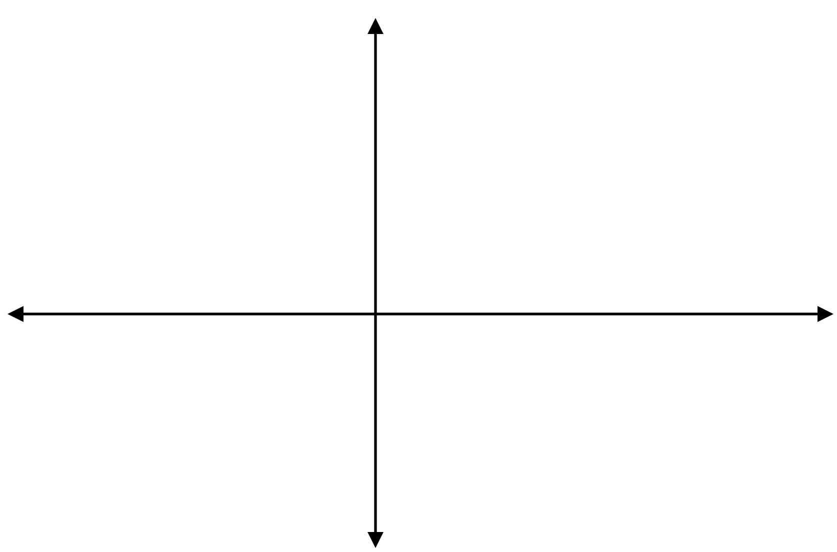
FULL EXAMPLE OF FIRST DERIVATIVE ANALYSIS: Given the function $f(x) = x^3 - 3x^2 + 5$, do the following:

a) Find the derivative and locate all critical points, in (x,y)-coordinate form. Do you need to add any other values to this list for a full list of Partition Values?

b) Build a sign chart for $f'(x)$. There are various ways to do this, but what is most important is to provide as many details as possible. Select proper "test values" within each Interval and determine whether $f'(x)$ is Positive(indicating Increasing) or Negative(Indicating Decreasing)

c) Make lists of ALL intervals of Increase/Decrease, using Interval Notation

d) Name each critical point as a Local Max, Local Min, or Neither. Use of the First Derivative Test is recommended. Once this is done, make a "rough sketch" of the function on the axes below, plotting all of the CPs and then drawing a "smooth curve" through them.



EXERCISE: Use the process detailed on the previous page analyze 1st Derivative information on:

$$f(x) = 3x^4 - 4x^3 - 36x^2$$

a) Find $f'(x)$, set $f'(x) = 0$ and factor it to determine the CPs. What is the Partition Values list?
b) Build a Sign Chart, select Test Values and use them to determine all Intervals of Increase/Decrease
c) Decide whether each CP is a Relative Max, Relative Min, or Neither
d) Sketch a rough graph

EXERCISE: Use the process detailed on the previous page analyze 1st Derivative information on:

$$f(x) = 8 + 15x^3 - x^5$$

a) Find $f'(x)$, set $f'(x) = 0$ and factor it to determine the CPs. What is the Partition Values list?
b) Build a Sign Chart, select Test Values and use them to determine all Intervals of Increase/Decrease
c) Decide whether each CP is a Relative Max, Relative Min, or Neither
d) Sketch a rough graph

EXERCISE: Use the process detailed on the previous page analyze 1st Derivative information on:

$$f(x) = 3x^{5/3} - 60x^{2/3}$$

a) Find $f'(x)$, set $f'(x) = 0$ and factor it to determine the CPs. Are other values needed for the Partition Values list?
b) Build a Sign Chart, select Test Values and use them to determine all Intervals of Increase/Decrease
c) Decide whether each CP is a Relative Max, Relative Min, or Neither
d) Sketch a rough graph

EXERCISE: Use the process detailed on the previous page analyze 1st Derivative information on:

$$f(x) = x^3 e^{-x}$$

a) Find $f'(x)$, set $f'(x) = 0$ and factor it to determine the CPs. Are other values needed for the Partition Values list?
b) Build a Sign Chart, select Test Values and use them to determine all Intervals of Increase/Decrease
c) Decide whether each CP is a Relative Max, Relative Min, or Neither
d) Sketch a rough graph

SECTION 4.2: SECOND DERIVATIVE AND CONCAVITY/INFLECTION POINTS

The process described in lecture for analyzing information gathered from the 2nd Derivative should have seemed very similar to what was done in regards to the 1st Derivative, with a few details handled a bit differently like the naming of items and information. The main steps, however, are extremely similar(looking for where the derivative = 0 or doesn't exist, building a sign chart, categorizing intervals).

Given: $f(x) = x^4 - 24x^2 + 8x$

a) Find $f'(x)$ and then find $f''(x)$.

b) Set $f''(x) = 0$ and determine all "possible" Inflection points. They may or may not eventually be classified as Inf Pts, that is yet to be determined. For good practice, obtain the full coordinates of each.

c) Build a Sign Chart, similar to those done for the 1st Derivative. As always, decide if there are any Vertical Asymptotes that would also need to be included as Partition Values(this function has none). Select Test Values to decide whether each Interval is one where *f(x)* is Concave Up($f''(x)$ is Positive) or *f(x)* is Concave Down($f''(x)$ is Negative).

d) As described in lecture, any change in Concavity (Up to Down or Down to Up) at the possible Inflection Points is an indication that it is indeed an actual Inflection Point. Check for a change in Concavity at each of the points you found in part (b). Are they Inflection Points?

EXERCISE: Given $f(x) = x^5 - 10x^4 + 8x$

a) Find $f''(x)$ and set it = 0. Locate all possible Inflection Points, in Coordinate Form.

b) Build a Sign Chart to determine ALL Intervals of Concavity, specifying Up or Down

c) Determine which points in part (a) are actually Inflection Points

EXERCISE: Given $f(x) = 9x^{5/3} - 135x^{2/3}$

a) Find $f''(x)$ and set it = 0. Locate all possible Inflection Points, in X-Coordinates only.

b) Build a Sign Chart to determine ALL Intervals of Concavity, specifying Up or Down

c) Determine which points in part (a) are actually Inflection Points

EXERCISE: Given $f(x) = e^{-x^2}$

a) Find $f''(x)$ and set it = 0. Locate all possible Inflection Points, in X-Coordinates only.

b) Build a Sign Chart to determine ALL Intervals of Concavity, specifying Up or Down

c) Determine which points in part (a) are actually Inflection Points

EXERCISE: Given $f(x) = 12 + 10x^4 - x^5$, use 1st and 2nd Derivatives

a) Determine all Intervals where *f(x)* is Increasing/Decreasing

b) Determine whether each CP is a Relative Maximum, Relative Minimum, or Neither

c) Determine all Intervals of Concavity, specifying Up/Down appropriately

d) Determine whether each possible Inflection Pt is an actual one

Sketch a Continuous function exhibiting all of the following information.

a)
$$
\begin{aligned}
&f(-2) = -1 \ , \quad f(1) = 3 \\
&f'(-2) = DNE \\
&f'(x) > 0 \quad if \quad (-2, \infty) \\
&f'(x) < 0 \quad if \quad (-\infty, -2) \\
&f''(-2) = DNE \ , \ f''(1) = 0 \\
&f''(x) > 0 \ \ if \quad (-2, 1) \\
&f''(x) < 0 \ \ if \quad (-\infty, -2) \ , \ (1, \infty)
\end{aligned}
$$

b)
$$
\begin{aligned}
&f(-1) = 3 \ , \quad f(2) = 1 \\
&f'(-1) = 0 \ , \quad f'(2) = DNE \\
&f'(x) > 0 \quad if \quad (-\infty, -1) \ , \ (2, \infty) \\
&f'(x) < 0 \quad if \quad (-1, 2) \\
&f''(2) = DNE \\
&f''(x) > 0 \ \ if \quad (2, \infty) \\
&f''(x) < 0 \ \ if \quad (-\infty, 2)
\end{aligned}
$$