SECTION 2.1: LIMITS, PROPERTIES OF LIMITS

Use the Properties of Limits that were given in lecture and shown in the textbook to find the limit below.

$\lim\limits_{x\to 4}\left(x^3-7x^2+\sqrt{x}+11\right)$ First use the property: $\lim\limits_{x\to c}\left[f(x)\pm g(x)\right]=\lim\limits_{x\to c} f(x)\pm\lim\limits_{x\to c} g(x)$

and then continue using other properties until you have an answer.

Now use the Algebraic Limit Property, $\lim\limits_{x\to c} f(x)=f(c)$, to directly find the limit in the exercise above.

$\lim\limits_{x\to 4}\left(x^3-7x^2+\sqrt{x}+11\right)$

Continue using the Algebraic Property on the following limits:

a) $\lim\limits_{x\to 2}\left(4x^3-5x^2+6x-9\right)$

b) $\lim\limits_{x\to 16}\left(\sqrt[4]{x}+\sqrt{x}\right)$

c) $\lim\limits_{x\to 4}\dfrac{x+5}{x+2}$

d) $\lim\limits_{x\to 3}\dfrac{x+7}{x^2-4}$

All of them have Real Number answers, but is this always the case?

Try using the Algebraic Property on this Limit: $\lim\limits_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 4}$

What result do you get? Undefined? But quite specifically $\frac{0}{0}$, which we call "Indeterminate". First, use 1-Sided Limits to see if you can determine the correct value of the limit. Fill in both tables:
Keep several decimal places of accuracy, as needed.

$\lim\limits_{x \to 2^-} \frac{x^2 + 3x - 10}{x^2 - 4}$

x	1.0	1.5	1.9	1.99	1.999
f(x)					

$\lim\limits_{x \to 2^+} \frac{x^2 + 3x - 10}{x^2 - 4}$

x	3.0	2.5	2.1	2.01	2.001
f(x)					

Now that both tables are filled out, use them to predict the value of the 2-sided Limit. To verify our "guess", use some regular old Algebra to factor the function in the limit, then reduce appropriately, and then retry using the Algebraic Limit Property. Do you get the value you predicted?

Verify you would get $\frac{0}{0}$, and then use factoring on the following limits:

a) $\lim\limits_{x \to 5} \frac{2x - 10}{x^2 - 4x - 5}$

b) $\lim\limits_{x \to -2} \frac{x^2 + 6x + 8}{x^2 - 4}$

c) $\lim\limits_{x \to 3} \frac{x^2 - 6x + 9}{x^2 - 4x + 3}$

REMEMBER: getting $\frac{0}{0}$ doesn't mean you cannot still find a Real-Numbered limit

Now try to find this limit: $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = ?$ You once again get $\frac{0}{0}$, Indeterminate.

Recall from Algebra the Conjugate to a 2-term expression. Here, we can use the conjugate of the numerator, $\sqrt{x}+2$, multiplying it with both the numerator and denominator of the function in the limit. Do so, and then try to simplify. You should be able to reduce the fraction and then use the Algebra Property of Limits. What do you get? It should be lim = ¼

Exercises: Find the following limits by the various techniques seen in this worksheet

a) $\lim_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$

b) $\lim_{x \to -3} \left(x^3 - 5x + 7\right)$

c) $\lim_{x \to 2} \frac{x^3 - 4x}{x^2 - 4}$

d) $\lim_{x \to 6} \frac{3x+4}{x-2}$

e) $\lim_{x \to 2} (2x+3)(x-4)$

f) $\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^2 - 3x - 4}$

SECTION 2.2: INFINITE LIMITS AND LIMITS AT INFINITY

Build tables of values to help determine what is happening in each limiting situation.

$$\lim_{x \to 4^-} \frac{12}{x-4}$$

x	3.0	3.5	3.9	3.99	3.999
f(x)					

$$\lim_{x \to 4^+} \frac{12}{x-4}$$

x	5.0	4.5	4.1	4.01	4.001
f(x)					

Now, what do these results say about the 2-sided limit: $\lim_{x \to 4} \frac{12}{x-4} =$?

Try to determine what we know for each of these similar cases.

i) $\lim_{x \to 2} \frac{5}{(x-2)^2} = ?$

ii) $\lim_{x \to 3} \frac{1-x}{x-3}$

iii) $\lim_{x \to 4} \frac{x+3}{x^2-3x-4}$

Now suppose we wish to look at a Limit where x doesn't approach a specific value, but rather gets larger and larger in either the Positive or Negative direction, so $\lim_{x\to\infty} f(x)$

First, we want to take note of a limit situation that is quite useful for certain limits as $x \to \infty$, which states the following: $\lim_{x\to\pm\infty} \frac{k}{x^p} = 0$ so long a k is any constant and $p > 0$.

For example, find: $\lim_{x\to\infty} \frac{2x^2+9}{3x^2-4x}$. This must be done by proper techniques in Calculus, not the short-cuts you should have learned in Algebra. To later use the property above, we look for the highest exponent in the Denominator of the function in our Limit, here a 2. We then multiply both the Numerator and Denominator of the function by $1/x^2$ and simplify each term as much as possible. So, we would see this:

$$\lim_{x\to\infty} \frac{\left(2x^2+9\right)}{\left(3x^2-4x\right)} \cdot \frac{1/x^2}{1/x^2} = ?$$

Rewrite the Limit, and then apply the property accordingly.

What value do you get?

Try a similar approach to the following limits:

i) $\lim_{x\to\infty} \frac{7x^2+2x}{5x^3-4x}$

ii) $\lim_{x\to\infty} \frac{8x^3+3x^2-11}{6x^3-4}$

iii) $\lim_{x\to\infty} \frac{2x+9}{3x-4x^2}$

iv) $\lim_{x\to\infty} \frac{2x^4+9}{3x-4x^3}$

SECTION 2.3: CONTINUITY

The Definition for Continuity of a function f(x) at a specific point $x = c$ states that f(x) is Continuous at c if three items are satisfied:

i) f(c) Exists , ii) $\lim_{x\to c} f(x)$ Exists , and iii) $f(c) = \lim_{x\to c} f(x)$ (that the first 2 items agree with each other)

Use the Definition, item-by-item, to determine whether each function, f(x), is Continuous at the specified value $x = c$

a) $f(x) = x^2 + 4x$ at $x = 3$

i) ? ii) ? iii) ?

b) $f(x) = \dfrac{x^2 - 4x}{x - 4}$ at $x = 4$

i) ? ii) ? iii) ?

c) $f(x) = \begin{cases} \dfrac{x^2 - 4x}{x - 4} & if \quad x \neq 4 \\ 2 & if \quad x = 4 \end{cases}$ at $x = 4$

i) ? ii) ? iii) ?

d) $f(x) = \begin{cases} x^2 - 9 & if \quad x < 4 \\ x + 3 & if \quad x \geq 4 \end{cases}$ at x =4

e) $f(x) = \begin{cases} x^2 - 6 & if \quad x < 4 \\ x + 3 & if \quad x \geq 4 \end{cases}$ at x =4

SECTION 2.4: DEFINITION OF THE DERIVATIVE

The Average Rate of Change of f(x) on [a,b] was shown in class using the formula: $\frac{f(b)-f(a)}{b-a}$

Find the Average Rate of Change for each given f(x) on the intervals [a,b]

i) $f(x)=x^2-6x$ on: a) $[2,7]$ b) $[0,4]$ c) $[-1,3]$

ii) $f(x)=x^2+4x+6$ on: DO NOT ROUND YOUR ANSWERS

a) $[2,4]$

b) $[2,2.5]$

c) $[2,2.1]$

d) $[2,2.01]$

e) $[2,2.001]$

After finding all 4 Avg Rates above, try to predict what the values are "headed towards" as the Interval shrinks towards a width of 0.

Now, let's use the Definition of the Derivative: $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Suppose $f(x)=x^2+4x+6$.

The first(and maybe most difficult) step is to properly construct and simplify $f(x+h)$

We look again at f(x), which tells us: $f(x+h)=(\qquad)^2+4(\qquad)+6$ where what goes in each ( )?
Fill them in and then simplify.

Now incorporate the simplified f(x+h) into the full formula: $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

i)The fraction should simplify using careful Algebra.
ii)Eventually, you should see that a factor of *h* can be
factored away from each term in the Numerator. Do so.
iii) Only now should you "cancel" that *h*, and then take
the Algebraic Limit by "plugging in 0 for *h*"

Did you get $f'(x)=2x+4$???

Use the Definition on each of the following functions:

a) $f(x) = 3x - 5x^2$

b) $f(x) = 4\sqrt{x}$

c) $f(x) = \dfrac{6}{2x+3}$