Math 165
Test 2
Review Questions

1. Use the first derivative test to identify all relative minimums and maximums. Be sure to identify the intervals where the function is increasing and decreasing.
    a. $f(x)=x^3+x^2-8x+5$
    b. $f(x)=\dfrac{x^2}{x-2}$
    c. $f(x)=3-\sqrt[3]{(x-5)^2}$

2. Use the second derivative test to identify all relative minimums and maximums. Be sure to identify the concavity at each critical point.
    a. $f(x)=-x^2+12x-9$
    b. $f(x)=3x^4-2x^3-12x^2+18x+3$

3. Find absolute minimum and absolute maximum for the function on the interval.
    a. $f(x)=x^2-6x-4$ on $[0,4]$
    b. $f(x)=-x^3-3x^2+9x+6$ on $[-4,2]$
    c. $f(x)=-4x^2+6x-9$ on $(-\infty,\infty)$

4. Evaluate the following limits:
    a. $\lim\limits_{x\to 1}\dfrac{\ln x}{x}$
    b. $\lim\limits_{x\to 1}\dfrac{x^3-1}{4x^3-x-3}$
    c. $\lim\limits_{x\to 1}\dfrac{x^2}{e^x-1-x}$

5. Find all points of inflection. Be sure to identify the intervals where the function is concave up and concave down.
    a. $f(x)=x^4-4x^3+10$
    b. $f(x)=3x^4-7x+1$
    c. $f(x)=2x^3-15x^2+12x+3$

6. Solve the following optimization questions.
   a. A company wants to construct an open box with a square base that has a volume of 32 cubic feet. How should the box be constructed to minimize surface area?
   b. A farmer wishes to build a pen adjacent to a river. He needs fencing for three sides. He has 240 feet of fencing. How should be build his fence in order to maximize the area of the pen?
   c. A company manufactures and sells x digital cameras per week. The weekly price-demand function is $p=400-0.4x$ and the weekly cost function is $C(x)=2000+160x$.
      i. How many items should be manufactured and sold to maximize revenue?
      ii. How many items should be manufactured and sold to maximize profit?

7. Find the following antiderivatives.
   a. $\int \left(5x^2+3x+1\right)dx$
   b. $\int \frac{5x^3+7x^2-3x+1}{2x}dx$
   c. $\int \left(4\sqrt{x}+\frac{9}{x^5}\right)dx$

8. Find the following antiderivatives.
   a. $\int 3x\left(7x^2+9\right)^5 dx$
   b. $\int \frac{9x}{5x^2+11}dx$
   c. $\int xe^{5x^2}dx$

9. Use four rectangles and left endpoints to approximate the following integrals.
   a. $\int_0^4 \left(x^2+1\right) dx$
   b. $\int_1^9 \left(x^2-20x-6\right) dx$
   c. $\int_0^2 3^x\, dx$

10. Evaluate 9a and 9b using the fundamental theorem of calculus. Evaluate 9c using the fundamental theorem and knowing $\int 3^x dx=\frac{3^x}{\ln 3}+C$

11. Evaluate the following definite integrals.

a. $\int_{1}^{4}\left(\frac{1}{x}-x^{2}\right)dx$

b. $\int_{0}^{1} 8x\left(x^{2}+1\right)^{3}\, dx$

c. $\int_{2}^{5}\frac{1}{\sqrt{6-t}}\,dt$

12. Find the area between the curves.

a. $f(x)=5-x^{2}$ and $g(x)=2-2x$

b. $y=x^{3}+1$ and $y=0$ for $0\le x\le 2$.

c. $f(x)=x^{2}$ and $g(x)=\sqrt{x}$