Math 165
Test 2
Review Questions

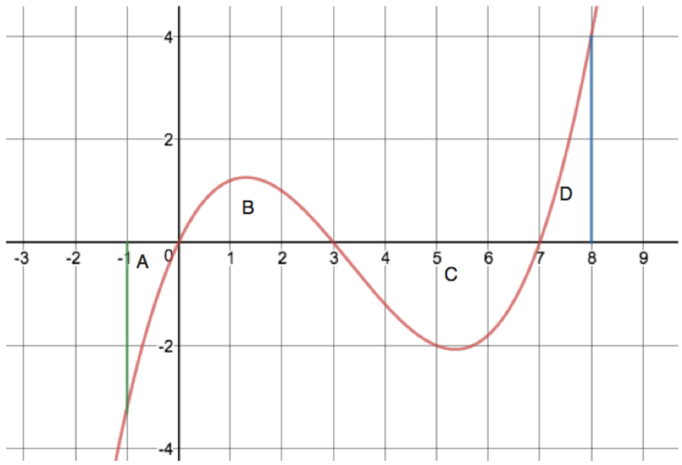
1. Use the first derivative test to identify all relative minimums and maximums. Be sure to identify the intervals where the function is increasing and decreasing.
   a. $f(x) = x^3 + x^2 - 8x + 5$
   b. $f(x) = \dfrac{x^2}{x-2}$
   c. $f(x) = 3 - \sqrt[3]{(x-5)^2}$

2. Use the second derivative test to identify all relative minimums and maximums. Be sure to identify the concavity at each critical point.
   a. $f(x) = -x^2 + 12x - 9$
   b. $f(x) = 3x^4 - 2x^3 - 12x^2 + 18x + 3$

3. Find absolute minimum and absolute maximum for the function on the interval.
   a. $f(x) = x^2 - 6x - 4$ on $[0,4]$
   b. $f(x) = -x^3 - 3x^2 + 9x + 6$ on $[-4,2]$
   c. $f(x) = -4x^2 + 6x - 9$ on $(-\infty,\infty)$

4. Evaluate the following limits:
   a. $\lim_{x\to 1} \dfrac{\ln x}{x}$
   b. $\lim_{x\to 1} \dfrac{x^3 - 1}{4x^3 - x - 3}$
   c. $\lim_{x\to 0} \dfrac{x^2}{e^x - 1 - x}$
   d. $\lim_{x\to -1} \dfrac{x^2 + 5x + 4}{x^3 + 1}$
   e. $\lim_{x\to -2} \dfrac{x^2 + 2x + 1}{x^2 + x + 1}$

5. Given the following graph of the function $y = f(x)$.



The area of region A is 1.41
The area of region B is 2.48
The area of region C is 5.33
The area of region D is 1.79

Evaluate the following integrals

a. $\int_{-1}^{8} f(x)dx$

b. $\int_{-1}^{3} f(x)dx$

c. $\int_{0}^{-1} f(x)dx$

d. $\int_{3}^{0} f(x)dx$

e. $\int_{2}^{2} f(x)dx$

f. Approximate $\int_{5}^{6} f(x)dx$

6. Solve the following optimization questions.
    a. A company wants to construct an open box with a square base that has a volume of 32 cubic feet. How should the box be constructed to minimize surface area?
    b. A farmer wishes to build a pen adjacent to a river. He needs fencing for three sides. He has 240 feet of fencing. How should be build his fence in order to maximize the area of the pen?
    c. A company manufactures and sells x digital cameras per week. The weekly price-demand function is $p = 400 - 0.4x$ and the weekly cost function is $C(x) = 2000 + 160x$.
        i. How many items should be manufactured and sold to maximize revenue?
        ii. How many items should be manufactured and sold to maximize profit?
    d. A concert promoter believes that a ticket price of $90 will result in a demand of 2000 tickets. He believes that each price reduction of $5 will result in an additional demand of 250 tickets.
        i. How many price reductions will maximize revenue?
        ii. What is the price that the promoter should charge to maximize revenue?
        iii. What is the maximum revenue?

7. Find the following antiderivatives.
    a. $\int \left(5x^2 + 3x + 1\right) dx$
    b. $\int \frac{5x^3 + 7x^2 - 3x + 1}{2x} dx$
    c. $\int \left(4\sqrt{x} + \frac{9}{x^5}\right) dx$

8. Find the following antiderivatives.
    a. $\int 3x\left(7x^2 + 9\right)^5 dx$
    b. $\int \frac{9x}{5x^2 + 11} dx$
    c. $\int x e^{5x^2} dx$

9. Use four rectangles and left endpoints to approximate the following integrals.

a. $\int_0^4 \left(x^2+1\right) dx$

b. $\int_1^9 \left(x^2-20x-6\right) dx$

c. $\int_0^2 3^x \, dx$

10. Evaluate 9a and 9b using the fundamental theorem of calculus. Evaluate 9c using the fundamental theorem and knowing $\int 3^x dx = \frac{3^x}{\ln 3} + C$

11. Evaluate the following definite integrals.

a. $\int_1^4 \left(\frac{1}{x} - x^2\right) dx$

b. $\int_0^1 8x\left(x^2+1\right)^3 dx$

c. $\int_2^5 \frac{1}{\sqrt{6-t}} dt$

12. Find the area between the curves.

a. $f(x) = 5 - x^2$ and $g(x) = 2 - 2x$

b. $y = x^3 + 1$ and $y = 0$ for $0 \le x \le 2$.

c. $f(x) = x^2$ and $g(x) = \sqrt{x}$

13. Evaluate the following integrals:

a. $\int \left(4x^2 + \frac{5}{x} + 7\sqrt{x} + 2\right) dx$

b. $\int e^{7x} \, dx$

c. $\int 5x e^{-x^2} \, dx$

d. $\int \frac{9x^2}{\sqrt[3]{2x^3+11}} dx$

14. Solve the following differential equations.

a. $\frac{dy}{dx} = 5x$

b. $\frac{dy}{dx} = 0.2y, \quad y(0) = 2$

c. $\frac{dA}{dt} = 0.025A, \quad A(0) = 500$