Math 180 **Name (Print):** ______________________

Final Exam **UIN:** ______________________

12/10/2015 **UIC Email:** ______________________
Time Limit: 2 Hours

This exam contains 12 pages (including this cover page) and 13 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on the top of this page.

The following rules apply:

- You may not open this exam until you are instructed to do so.
- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!
- CHECK THAT THE NUMBER ON TOP OF EACH PAGE IS THE SAME! IF THEY ARE NOT THEN NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY!

TA Name: ______________________

Circle your instructor.

- Bode
- Goldbring
- Hachtman
- Riedl
- Sinapova
- Steenbergen @ 11am
- Steenbergen @ 12pm
- Steenbergen @ 2pm

1. (14 points) Suppose that f is a continuous function. True, False or not enough information given? Circle the correct answer.

Consider the following table.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
-1	2	0	2	*unknown*
2	8	0	-1	0
4	7	1	0	0

(I) f has a local minimum at $x = -1$.

True **False** **Not enough information given**

(II) f has a local minimum at $x = 2$.

True **False** **Not enough information given**

(III) f has a point of inflection at $x = 4$.

True **False** **Not enough information given**

(IV) There is a c in the interval $[-1, 4]$ with $f(c) = 5$.

True **False** **Not enough information given**

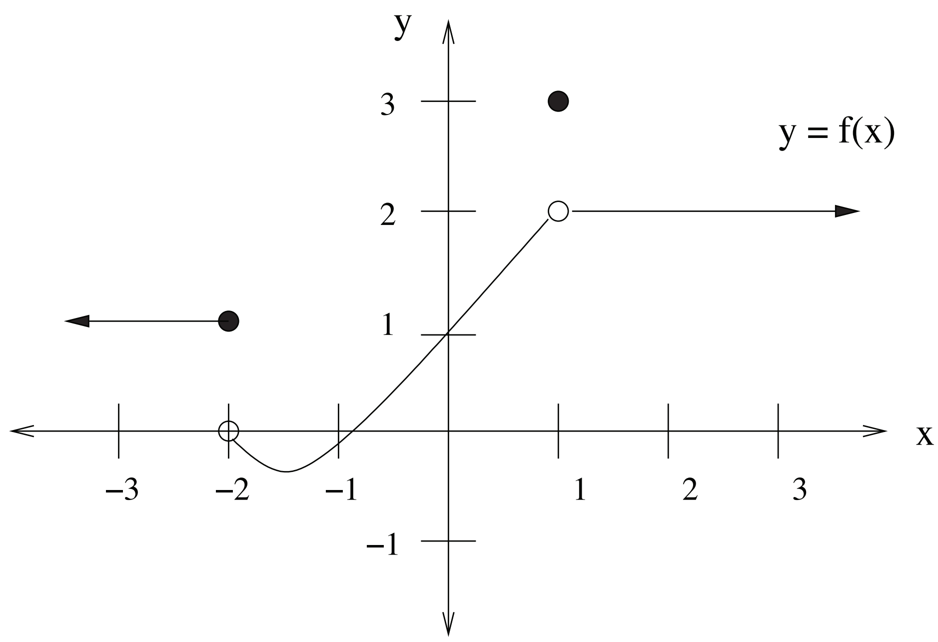
(V) There is some point d with $-1 < d < 4$ where $f'(d) = 1$.

True **False** **Not enough information given**

(VI) $y = x + 3$ is tangent to the graph of f at $x = 4$.

True **False** **Not enough information given**

2. (12 points) Consider the function $f(x)$ whose graph is given below.



1. (4 points) Find $\lim_{x \to -2} f(x)$ or explain why it does not exist.

$\lim_{x\to -2^-} f(x) = 1$

$\lim_{x\to -2^+} f(x) = 0$

$\lim_{x\to 2} f(x)$ does not exist. "jump"

2. (2 points) Find $\lim_{x \to 0} f(x)$ or explain why it does not exist.

$\lim_{x\to 0} f(x) = 1$

3. (3 points) Find $\lim_{x \to 1} f(x)$ or explain why it does not exist.

$\lim_{x\to 1} f(x) = 2$

4. (3 points) Is f continuous at $x = 1$? Explain why or why not.

No, because

$3 = f(1) \neq \lim_{x\to 1} f(x) = 2$

3. (11 points) Evaluate the following limits. Make sure to state all theorems that you are using.

(a) (5 points) Assume $4 \leq f(x) \leq 5$ for all x. Evaluate $\lim_{x\to 0} x f(x)$ or state that it does not exist (DNE). Justify your answer, and state all theorems that you are using.

By the squeeze theorem

$$4x \leq x f(x) \leq 5x$$

Thus

$$0 = \lim_{x\to 0} 4x \leq \lim_{x\to 0} x f(x) \leq \lim_{x\to 0} 5x = 0$$

$$\Rightarrow \boxed{\lim_{x\to 0} x f(x) = 0}$$

(b) (6 points) $\lim_{x\to 0} \frac{x^2}{1-\cos x}$

of type $\frac{0}{0}$ use L'Hôpital

$$\underset{L'H}{=} \lim_{x\to 0} \frac{2x}{\sin x} \quad \text{of type } \frac{0}{0}$$

$$\underset{L'H}{=} \lim_{x\to 0} \frac{2}{\cos x} = \boxed{2}$$

4. (14 points) Evaluate the following limits. Make sure to state all theorems that you are using.

(a) (6 points) $\lim_{x\to 2} \frac{x^2-4}{1+\sin(x-2)}$ $= \frac{0}{1} = 0$

(b) (8 points) $\lim_{x\to 0} \left(1+2x\right)^{\frac{1}{x}}$ $= L$

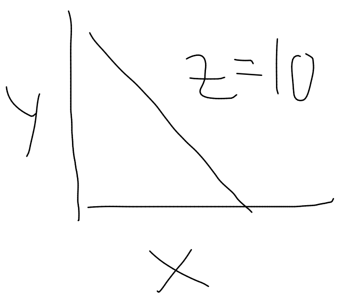
$\ln L = \lim_{x\to 0} \frac{1}{x} \ln(1+2x)$

$= \lim_{x\to 0} \frac{\ln(1+2x)}{x}$ of type $\frac{0}{0}$

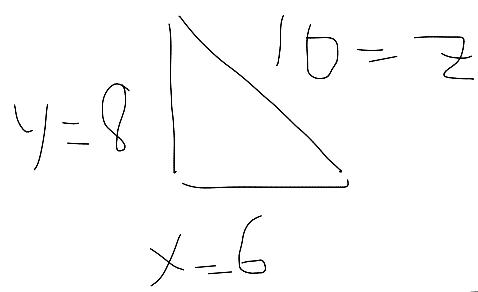
$\underset{L'H}{=} \lim_{x\to 0} \frac{\frac{2}{1+2x}}{1} = 2$

$\Rightarrow$ limit $L = e^2$

5. (12 points) A 10-foot-long ladder resting against a vertical wall begins to slide away from it. If the bottom of the ladder slides at a rate of 2 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?



Know $\frac{dx}{dt} = 2$ Want $\frac{dy}{dt}$ when $x = 6$



$x^2 + y^2 = 10$

$\cancel{2} x x' + \cancel{2} y y' = 0$

$6 \cdot 2 + 8 \cdot y' = 0$

$\Rightarrow y' = -\frac{12}{8} = \boxed{-\frac{3}{2} \frac{ft}{sec}}$

6. (12 points) Let $f(x) = x^3 - 3x^2 - 9x + 1$. Find the absolute maximum and absolute minimum values of $f(x)$ over the interval $[-2, 2]$.

$f' = 3x^2 - 6x - 9 = 3(x-3)(x+1)$

$f' = 0$ $\cancel{x = 3}$ not in interval

$x = -1$

$f' = \emptyset$ N/A

endpts

	f
-2	-1
-1	6 = abs max
2	-21 = abs min

7. (14 points) At a certain gas station in Evanston, $f(x)$ is the number of gallons of gas sold during the day if the price per gallon is x dollars. It is known that $f(4) = 500$ and $f'(4) = -80$. The total revenue $R(x)$ earned by selling the gas during the day is $R(x) = xf(x)$.

(a) (7 points) Approximate $f(4.1)$ using a linear approximation of the function $f(x)$ at $a = 4$.

$$L(x) = f(4) + f'(4)(x-4)$$
$$= 500 - 80(x-4)$$
$$f(4.1) \approx 500 - 80(4.1-4)$$
$$= 500 - 8$$
$$= 492$$

(b) (7 points) Find $R'(x)$ when $x = 4$.

use product rule

$$R'(x) = 1 \cdot f(x) + x \cdot f'(x)$$
$$= 500 + 4 \cdot (-80)$$
$$= 180$$

8. (24 points) Find the **derivatives** of the following functions. You **do not need to simplify** your answers, but you need to explain your reasoning.

   (a) (7 points) $f(x) = \dfrac{\tan^{-1}(2x)}{e^{3x}+1}$ quotient + chain rule

   $$f' = \frac{\frac{2}{1+(2x)^2}\cdot(e^{3x}+1) - \tan^{-1}(2x)\cdot 3e^{3x}}{(e^{3x}+1)^2}$$

   (b) (7 points) $g(x) = \displaystyle\int_0^{x^3} (\cos^{10} t)dt$

   $f(u) = \int_0^u \cos^{10} t \, dt$

   $g'(x) = f'(x^3) \cdot 3x^2$ FTC + chain rule

   $= \cos^{10}(x^3) \cdot 3x^2$

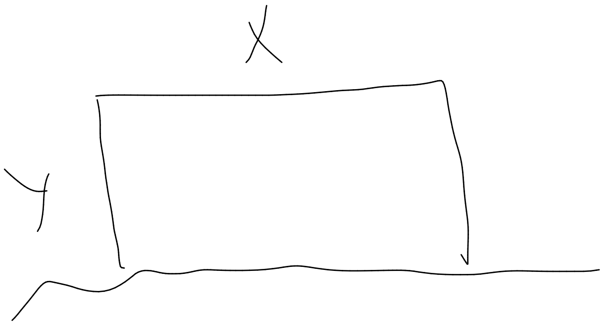
   (c) (10 points) $h(x) = x^{\tan(2x)}$ log diff

   $\ln(h(x)) = \tan(2x) \cdot \ln x$

   $\frac{1}{h(x)} \cdot h'(x) = 2\sec^2(2x) \cdot \ln x + \tan(2x) \cdot \frac{1}{x}$

   $h'(x) = \left(2\sec^2(2x)\ln x + \frac{\tan(2x)}{x}\right) \cdot x^{\tan(2x)}$

9. (15 points) A rectangular region is to be enclosed on three sides by a fence, and on the fourth side by a straight river. The enclosed area is to equal 3200 square feet. The cost of the fence is \$2 per ft. Find the dimensions that minimize the cost of the fence. Be sure to explain why your answer minimizes cost.



Minimize cost $= 2x + 4y$

subject to $xy = 3200 \Rightarrow y = \frac{3200}{x}$

$\Rightarrow f(x) = 2x + \frac{4 \cdot 3200}{x}$ Domain $= (0, \infty)$

$f'(x) = 2 - \frac{4 \cdot 3200}{x^2} = 0$

$x^2 = 2 \cdot 3200 = 6400$

$\Rightarrow x = 80$ & $y = 40$

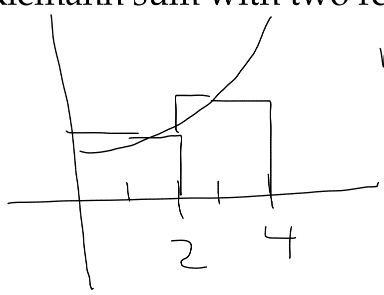
sign of f': $(0 \quad - \quad 80 \quad +$

$\searrow$ $\nearrow$

absolute min

$\Rightarrow$ dimension 80×40

10. (6 points) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = 0$ to $x = 4$ using a midpoint Riemann sum with two rectangles.



midpoints $x=1$ & $x=3$

$M_2 = f(1)\cdot 2 + f(3)\cdot 2$

$= 2\cdot 2 + 10\cdot 2$

$= \boxed{24}$

11. (15 points) Let $f(x)$ be an even continuous function, and let $g(x)$ be an odd continuous function. Suppose that

$$\int_0^8 f(x)\,dx = 9 \text{ and } \int_0^8 g(x)\,dx = -13.$$

Compute the following integrals:

(a)(5 points) $\int_0^8 (3f(x) + 2g(x))\,dx$.

$= 3\cdot 9 + 2(-13) = \boxed{1}$

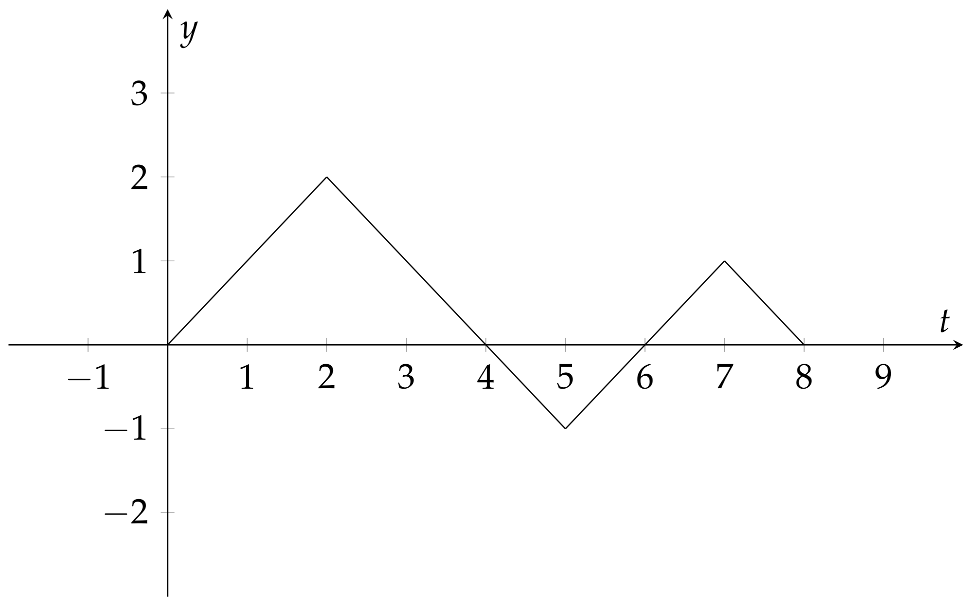
(b)(10 points) $\int_{-8}^8 (f(x) - 5g(x))\,dx$.

$= 2\int_0^8 f(x)\,dx - 5\int_{-8}^{8} g(x)\,dx$

$= 2\cdot 9 - 0$

$= 18$

12. (35 points) The function $g(x)$ is defined as $g(x) = \int_0^x f(t)\,dt$. The graph of the function $f(t)$ is given below.



(a) (10 points) Calculate $g(0), g(2), g(4), g(6)$ and $g(8)$.

$g(0) = 0$ $g(4) = 4$ $g(8) = 4$

$g(2) = 2$ $g(6) = 3$

(b) (2 points) Find the average value of f on the interval $[0, 8]$.

$\frac{1}{8-0}\int_0^8 f(t)\,dt = \frac{4}{8} = \frac{1}{2}$

(c) (4 points) Evaluate $g'(2)$. State what theorem you are using.

FTC $\Rightarrow g'(2) = f(2) = 2$

(d) (6 points) On what intervals is g increasing, and on what intervals is g decreasing?

increasing $(0,4)$ $(6,8)$

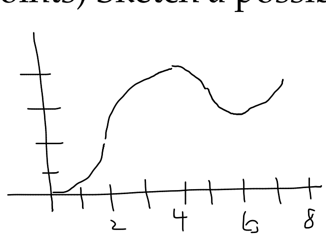
decreasing $(4,6)$

(e) (8 points) On what intervals is g concave up, and on what intervals is g concave down?

concave up $(0,2)$ $(5,7)$

concave down $(2,5)$ $(7,8)$

(f) (5 points) Sketch a possible graph for $g(x)$.



13. (16 points) Evaluate the following integrals.

(a) (8 points) $\int_0^1 \left(x^2 + \frac{1}{x^2+1}\right) dx$ (simplify your answer, your final answer should be a number!)

$$= \frac{x^3}{3} + \tan^{-1} x \Big|_0^1$$

$$= \frac{1}{3} + \tan^{-1}(1) - \left(\frac{0}{3} + \tan^{-1} 0\right)$$

$$= \frac{1}{3} + \frac{\pi}{4} = \frac{4+3\pi}{12}$$

(b) (8 points) $\int \sin x(\cos x + 2)^2 \, dx$

u-substitution

$$\boxed{\begin{aligned} u &= \cos x + 2 \\ du &= -\sin x \, dx \end{aligned}}$$

$$= -\int u^2 \, du = -\frac{u^3}{3} + c$$

$$= -\frac{(\cos x+2)^3}{3} + c$$