MATH 180 Exam 1

October 2, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 8 pages (including this cover page) and 10 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

TA Name:__________________________

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Martina Bode
- Mercer (Tabes) Bridges
- Nathan Jones
- Matthew Lee
- John Steenbergen

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1. (8 points) Suppose that $f(t)$ is the piecewise function

$$f(t) = \begin{cases} t - \cos\left(\dfrac{\pi t}{3}\right) & t < 3 \\ 3 & t = 3 \\ t^2 - 5 & t > 3 \end{cases}$$

(a) Find $\lim\limits_{t \to 3^-} f(t)$.

(b) Find $\lim\limits_{t \to 3^+} f(t)$.

(c) Find $\lim\limits_{t \to 3} f(t)$. Justify your answer!

(d) Is $f(t)$ continuous at $t = 3$? Why or why not?

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2. (21 points) Evaluate the following limits, and then answer the questions about asymptotes in part (d). If the limit is infinite, state whether it is ∞ or $-\infty$. Justify your answers.

   (a) (5 points) $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$

   (b) (5 points) $\lim_{x\to \infty} \dfrac{3x^3-10x+11}{4x^3+10}$

   (c) (5 points) $\lim_{x\to 2^+} \dfrac{x^2-3x+1}{x-2}$

   (d) (6 points) Let $f(x) = \dfrac{x^2+x-6}{x-2}$, $g(x) = \dfrac{3x^3-10x+11}{4x^3+10}$, and $h(x) = \dfrac{x^2-3x+1}{x-2}$ be the functions from above.
   Based on your answers in parts (a)-(c),

   (i) which of the functions f, g, and h have a vertical asymptote at $x = 2$?

   (ii) which of the functions f, g, and h have a horizontal asymptote?

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3. (12 points) Use the **limit definition** of the derivative to compute $f'(3)$ where

$$f(x) = \sqrt{x+6}$$

4. (6 points) Use the squeeze theorem to evaluate:

$$\lim_{x \to 0} \left[x^2 \sin\left(\frac{1}{x^2}\right) \right]$$

Remember to show all your work!

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5. (8 points) Consider the following table, which contains the values of three functions f, g, and h (as well as their derivatives) at two points:

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
2	4	1	3	-1	-3	2
3	-1	-4	1	10	5	-2

Compute the following derivatives:

(a) $\dfrac{d}{dx}\left([f(x)]^5\right)$ at $x = 3$.

(b) $\dfrac{d}{dx}\left(g(x)/h(x)\right)$ at $x = 2$.

6. (18 points) Find the derivatives of the following functions. You do not need to simplify your answers.

(a) $f(x) = 5x^3 + \dfrac{1}{4}\sin(x) + 6$

(b) $g(x) = e^{2x}\tan(3x)$

(c) $f(x) = \cos(\sqrt{x})$

7. (8 points) Suppose an ant is moving along a straight path, and $f(t)$ gives its position at time t, $t > 0$. Units are in ft and seconds. Suppose that you are given that the velocity function for the ant is $f'(t) = t^2 - 2t$.

   (a) For what value(s) of t does the ant have velocity 3 ft/s?

   (b) Find the acceleration of the ant.

8. (5 points) True or False. If false, sketch a graph or give a formula for a counter example.

   If f is any function with $f(-1) = -1$ and $f(1) = 1$, then there is a point c, such that $-1 < c < 1$ and $f(c) = 0$.

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9. (6 points) Assume that a function satisfies $f(1) = 3$, and $f'(1) = -4$. Find the equation of the tangent line to $y = f(x)$ at $x = 1$.

10. (8 points) The graph of the function $f(x)$ is shown below. Sketch the graph of the derivative function $f'(x)$.