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MATH 180 Exam 1
February 14, 2017

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ANSWER KEY

University Email: ____________________

UIN: ____________________

Circle your instructor's name: Shulman Steenbergen Thulin Zhang

TA's Name: ____________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (20 points) Find the following derivatives. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $\frac{d}{dx}\left(-4x^2 + x^{-10} + 5e^{-2x}\right)$

$= -8x - 10x^{-11} - 10e^{-2x}$

(power rule) (chain rule)

(b) $\frac{d}{dx}\left(3x^2 + x\tan x\right)$

$= 6x + (1 \cdot \tan x + x \cdot \sec^2 x)$

(product rule)

(c) $\frac{d}{dx}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ quotient rule

$u = \sqrt{x} - 1$ $\quad v = \sqrt{x} + 1$

$u' = \frac{1}{2\sqrt{x}}$ $\quad v' = \frac{1}{2\sqrt{x}}$

$$= \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}+1) - (\sqrt{x}-1)\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}$$

(d) $\frac{d}{dx}\left(\frac{8}{4+x^2}\right)$ quotient rule

$u = 8$ $\quad v = 4 + x^2$

$u' = 0$ $\quad v' = 2x$

$$= \frac{0 - 16x}{(4+x^2)^2}$$

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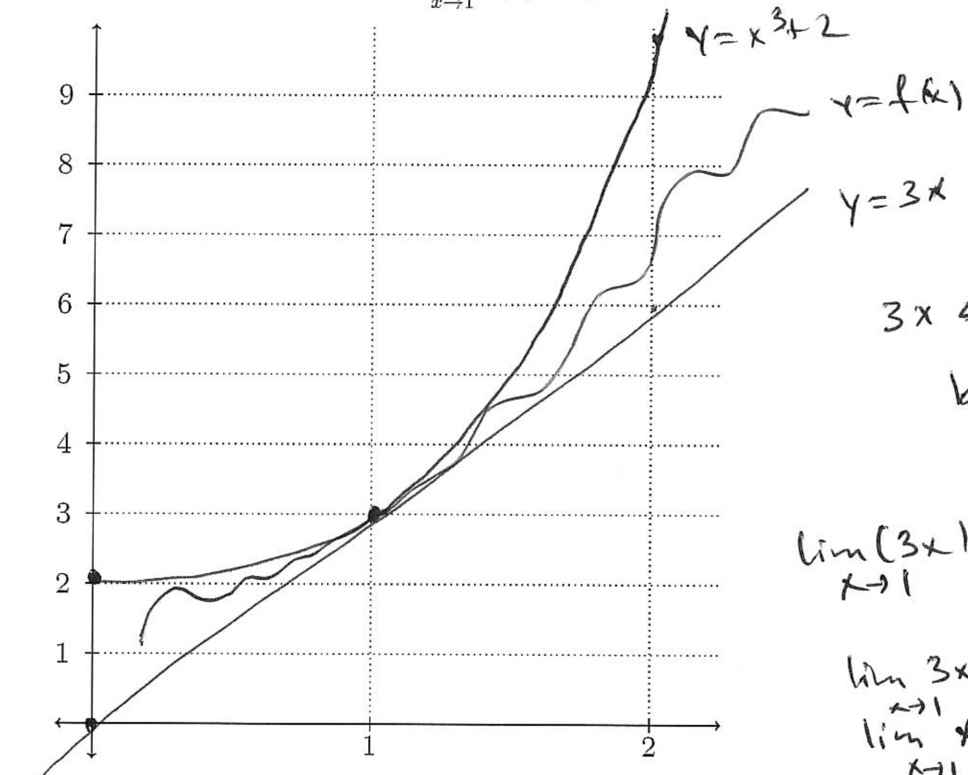
2. (8 points) Consider a function $f(x)$ such that $3x \le f(x) \le x^3 + 2$ for $0 \le x \le 2$. On the axes provided, sketch $y = 3x$ and $y = x^3 + 2$, and a possible graph for $f(x)$. Make sure you label each of the functions. Then calculate $\lim_{x\to 1} f(x)$. Explain your answer.



$3x \le f(x) \le x^3+2$

by squeeze theorem

$\lim_{x\to 1}(3x) \le \lim_{x\to 1} f(x) \le \lim_{x\to 1} (x^3+2)$

$\lim_{x\to 1} 3x = 3$

$\lim_{x\to 1} x^3+2 = 3$

$\Rightarrow \boxed{\lim_{x\to 1} f(x) = 3}$

3. (7 points) Calculate the following limit. Show all of your work.

$$\lim_{x\to-\infty} \frac{3x^2+7x-9}{x^2+2}$$

$$= \lim_{x\to-\infty} \frac{3 + \frac{7}{x} - \frac{9}{x^2}}{1 + \frac{2}{x^2}} = \boxed{3}$$

$\frac{7}{x} \to 0$, $\frac{9}{x^2} \to 0$, $\frac{2}{x^2} \to 0$

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4. (18 points) Calculate the following limits. If they exist, state their value. Otherwise, state the the limit is $+\infty$, $-\infty$, or does not exist.

(a) $\lim_{x \to 2^+} \frac{x-3}{x-2} = \boxed{-\infty}$ of type $\frac{-1}{0^+}$

Since $x \to 2^+$

$x > 2$

$x - 2 > 0$

(b) $\lim_{t \to -3} \frac{t^2+4t-3}{t^2-3} = \frac{(-3)^2+4(-3)-3}{(-3)^2-3} = \frac{-6}{6} = \boxed{-1}$

(c) $\lim_{x \to \infty} \left(\frac{1}{x^2} + \arctan x\right) = 0 + \frac{\pi}{2} = \boxed{\frac{\pi}{2}}$

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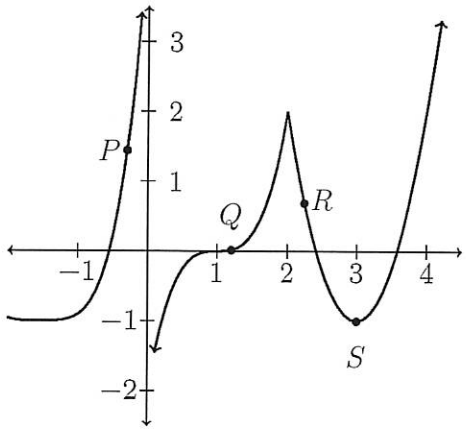
5. (8 points) Use the following graph to answer the questions below.



(a) Order the points P, Q, R, S in order from smallest to largest in terms of the derivative at each point.

$\underbrace{R}_{\text{slope negative}} < \underbrace{S}_{\text{slope is zero}} < \underbrace{Q}_{\text{slope is} > 0} < \underbrace{P}_{\text{slope is} > 0}$

& steeper than @ Q

(b) List all the x-values where the derivative does not exist.

$x = 0$ f is undefined @ $x = 0$

$x = 2$ corner point at $x = 2$

6. (8 points) If $f(x) = 4xe^{-x} + 2$, write the equation of the tangent line to $f(x)$ at $x = 0$.

$f'(x) = 4e^{-x} + 4x(-e^{-x})$

$f'(0) = 4 + 0 = 4$ $\Rightarrow$ slope $= 4$

$f(0) = 0 + 2 = 2$ $\Rightarrow$ point $= (0, 2)$

$\Rightarrow y - 2 = 4x$

$\Rightarrow \boxed{y = 4x + 2}$

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7. (12 points) The position of an object is given by $s(t) = -16t^2 + 96t + 200$ where s is in meters and t is in seconds.

   (a) Calculate the initial velocity of the object. Make sure to include your units.

   $v(t) = -32t + 96$

   $v(0) = 96$ ft/s

   (b) At what time (if any) is the object's velocity equal to 0?

   $v = -32t + 96 = 0$

   $t = \frac{96}{32} = 3$ seconds

   (c) Is the acceleration of the object the same for all time $t \geq 0$? Explain your answer.

   $a(t) = -32$ ft/s²

   yes, the acceleration is always -32 ft/s².

8. (9 points) Using the **definition of the derivative**, find $f'(x)$ if $f(x) = \frac{1}{x+2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \left(\frac{\cancel{x+2} - (\cancel{x}+h+\cancel{2})}{(x+h+2)(x+2)} \right)$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{-\cancel{h}}{(x+h+2)(x+2)}$$

$$= \boxed{\frac{-1}{(x+2)^2}}$$

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9. (10 points) For each statement below, CLEARLY either circle "T" for TRUE or "F" for FALSE (if it is not clear which one you chose, it will be marked wrong). You do not need to justify your answer.

(a) T or F: $x = 1$ is a vertical asymptote of $y = \dfrac{x^2 - 7x + 6}{x^2 - 1}$ $= \dfrac{(x-6)(x-1)}{(x+1)(x-1)}$

False b/c $\lim_{x \to 1} f(x) = \lim_{x \to 1} \dfrac{x-6}{x+1} = \dfrac{-5}{2}$

$\Rightarrow$ removable discontinuity

(b) T or F: If $f(x)$ is continuous at $x = 2$, then $f(x)$ is differentiable at $x = 2$.

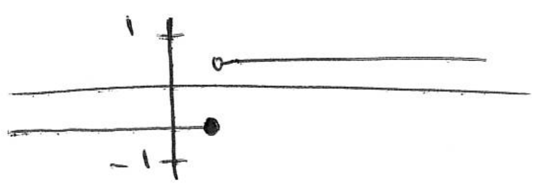
False for ex corner points are continuous but not differentiable.

(c) T or F: If $\lim_{x \to 3} f(x) = \infty$, then $\lim_{x \to 3^+} f(x) = \infty$.

True

(d) T or F: Every function with domain $(-\infty, \infty)$ and range $[-1, 1]$ is continuous.

False



(e) T or F: If $f(x)$ is continuous on $[0, 10]$ with $f(0) = 4$ and $f(10) = -11$, then $f(c) = 0$ for some c in $(0, 10)$.

True by Intermediate Value Theorem since 0 is between $f(0)=4$ & $f(10)=-11$ & since f is continuous on $[0,10]$.

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