MATH 180 Exam 2

October 31, 2017

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. This exam contains 8 pages (including this cover page) and 9 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

TA Name:____________________________

ANSWER KEY

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Martina Bode
- Jenny Ross
- Drew Shulman
- Matthew Woolf
- Sherwood Hachtman

DO NOT WRITE ABOVE THIS LINE!!

1. (16 points) Find the derivatives of the following functions. Use logarithmic differentiation if necessary.

(a) (6 points) $f(x) = \arctan(\ln(x^4 + 1))$

$$= \frac{1}{1+(\ln(x^4+1))^2} \cdot \frac{1}{x^4+1} \cdot 4x^3$$

(b) (10 points) $h(x) = (x^2 + 1)^{3x}$

logarithmic diff

$$\ln(h(x)) = 3x \cdot \ln(x^2+1)$$

$$\frac{1}{h(x)} \cdot h'(x) = 3 \cdot \ln(x^2+1) + 3x \cdot \frac{1}{x^2+1} \cdot 2x$$

$$h'(x) = \left(3\ln(x^2+1) + \frac{6x^2}{x^2+1}\right) \cdot (x^2+1)^{3x}$$

DO NOT WRITE ABOVE THIS LINE!!

2. (12 points) Find the equation of the tangent line to the curve $x^3y^2 + 2y = 8$ at $(1, 2)$.

implicit diff

$$3x^2y^2 + x^3 \cdot 2y \cdot \frac{dy}{dx} + 2\frac{dy}{dx} = 0$$

$x = 1 \quad y = 2$

$$3 \cdot (1)^2 \cdot (2)^2 + (1)^3 \cdot 2 \cdot 2 \cdot \frac{dy}{dx} + 2\frac{dy}{dx} = 0$$

$$12 + 6\frac{dy}{dx} = 0$$

$$6\frac{dy}{dx} = -12$$

$$\frac{dy}{dx} = -2 = \text{slope}$$

$$\Rightarrow \boxed{y - 2 = -2(x-1)} \quad \text{or} \quad \boxed{y = -2x + 4}$$

3. (4 points) The population of a haunted village is given by the function $P = f(t)$, t years since 2010. Suppose $P(6) = 400$ and $P'(6) = -10$.

(a) What are the units of the function $P'(t)$?

population per year

(b) Describe in words the meaning of $P'(6) = -10$.

At/After the 6th year the population decreases at a rate of 10 people per year.

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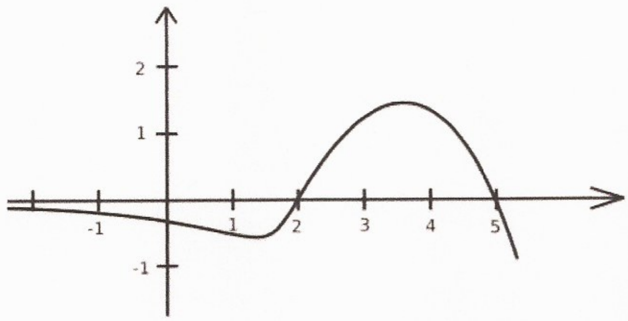
4. (10 points) The graph of a **derivative function** $f'(x)$ is shown below.



(a) On what intervals is f increasing or decreasing?

sign of f' $\quad - \quad 2 \quad + \quad 5 \quad -$

f is decreasing on $(-\infty, 2) \cup (5, \infty)$, increasing on $(2, 5)$

(b) At what values of x does f have a local maximum or minimum?

local min @ $x = 2$

local max @ $x = 5$

5. (10 points) Let $f(x) = xe^x$. On what intervals is f concave upward or downward?

$f'(x) = 1 \cdot e^x + x \cdot e^x = (x+1)e^x$

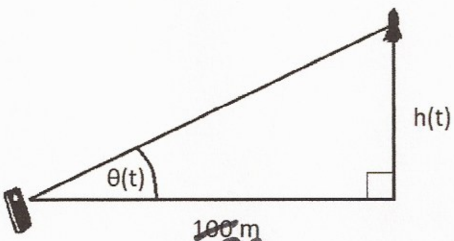
$f''(x) = 1 \cdot e^x + (x+1)e^x = (x+2)e^x$

sign of f'' $\quad - \quad -2 \quad +$

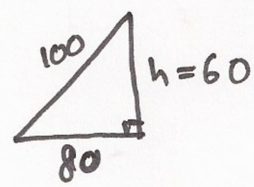
concave down on $(-\infty, -2)$

concave up on $(-2, \infty)$

6. (12 points) A rocket is launched directly upwards and moves at a constant speed of 20 meters per second. An observer is video recording the launch from ~~100~~ 80 meters away on her cell phone. To keep the rocket in the middle of the video, how fast should the angle of elevation of the cell phone be increasing when the rocket is 60 meters in the air? Write your answer as a reduced fraction and make sure to give proper units. (You can neglect the height of the person.)



Want: $\frac{d\theta}{dt}$ @ instant when



Given: $\frac{dh}{dt} = 20$ m/s

$\sec\theta = \frac{100}{80} = \frac{5}{4}$

$\tan\theta = \frac{h}{80}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{80} \cdot \frac{dh}{dt}$

$\left(\frac{5}{4}\right)^2 \cdot \frac{d\theta}{dt} = \frac{1}{80} \cdot 20$

$\frac{d\theta}{dt} = \frac{\cancel{16}^{\,4}}{25} \cdot \frac{1}{\cancel{4}} = \frac{4}{25}$ rad/sec

DO NOT WRITE ABOVE THIS LINE!!

7. (12 points) Find the absolute maximum and minimum of the function

$f(x) = \dfrac{x^2-5}{x+3}$ on the closed interval $[-2, 0]$.

$$f'(x) = \frac{(2x)(x+3) - (x^2-5)\cdot 1}{(x+3)^2}$$

$$= \frac{2x^2 + 6x - x^2 + 5}{(x+3)^2}$$

$$= \frac{x^2+6x+5}{(x+3)^2}$$

$f' = 0 \quad x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

$\cancel{x=-5} \quad x = -1$

$f' = \emptyset \quad \cancel{x = -3}$

Compare

-2	-1 = max value @ $x=-2$
-1	-2 = min value @ $x=-1$
0	$-5/3$

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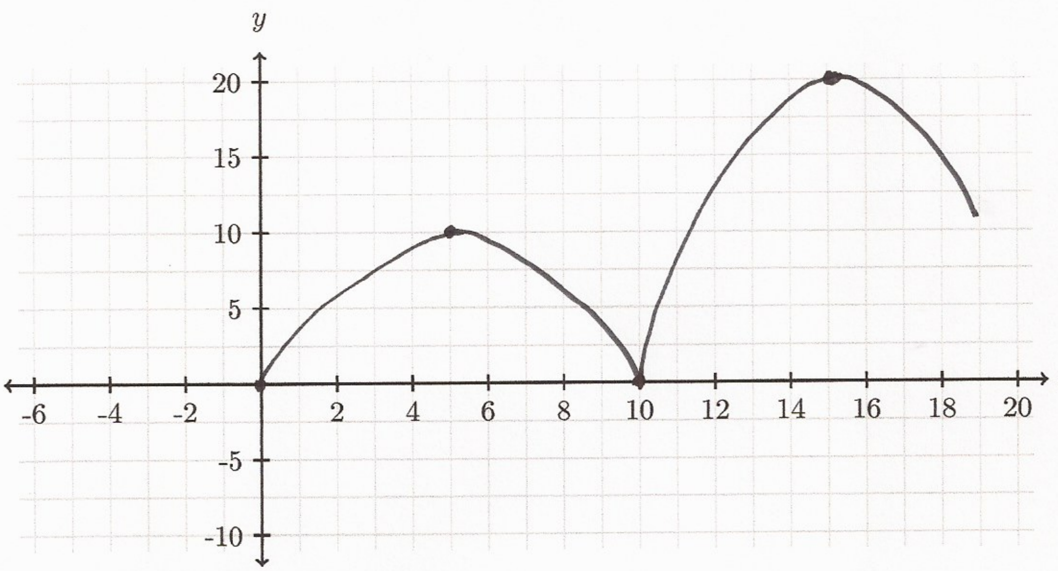
8. (10 points) Let $f(x)$ be a continuous function defined on the interval [0,20] with the following properties:

- $f(0) = 0,\ f(5) = 10,\ f(10) = 0,\ f(15) = 20.$
- $f'(x) > 0$ on $(-\infty, 5) \cup (10, 15)$; $f'(x) < 0$ on $(5, 10) \cup (15, \infty)$.
- $f''(x) < 0$ on $(-\infty, 10) \cup (10, \infty)$.

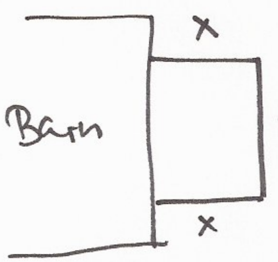
On the axes provided below, sketch a possible graph of this function.



9. (14 points) A rectangular pen is built with one side against a large barn. 400 feet of fencing will be used to enclose the rectangular area, with no fence along the side against the barn. Assume the barn is large enough to accommodate any size pen.

Find the dimensions of the pen that will enclose the largest area. To do so, answer the following.

(a) Sketch a diagram of the problem, define variables to be used and carefully label your picture.



(b) Find the objective (optimization) function. Include the domain.

Looking for max area $= x \cdot y$ subject to $2x + y = 400$

$y = 400 - 2x$

$\Rightarrow f(x) = x \cdot (400 - 2x)$

$= 400x - 2x^2$ Domain $= [0, 200]$

(c) Find the dimensions that enclose the largest area, and verify that your result gives the maximum possible area.

$f'(x) = 400 - 4x$ use closed interval method:

(i) $f' = 0$ $x = 100$

(ii) $f' = \phi$ N/A

(iii) endpoints $x=0$ & $x=200$

x	Area
0	0
100	$100 \cdot 200 = 20{,}000$
200	0

Dimensions are

$\boxed{x = 100 \times y = 200}$

or sign of f'

$+$ $-$

0 ↗ 100 ↘ 200

abs max @ $x = 100$