MATH 180 Exam 2
March 20, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 8 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

TA Name:________________________

Answers

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Martina Bode
- Gary (Clark) Alexander
- John Steenbergen
- Daniel Braithwaite

DO NOT WRITE ABOVE THIS LINE!!

1. (22 points) Differentiate the following functions, use logarithmic differentiation if needed. You do not need to simplify your answers.

(a) (6 points) $f(x) = \arcsin(x^8)$

$$f'(x) = \frac{1}{\sqrt{1-(x^8)^2}} \cdot 8x^7$$

(b) (6 points) $f(x) = \ln(x^2 - 4)$

$$f'(x) = \frac{2x}{x^2-4}$$

OR

$$f(x) = \ln(x+2) + \ln(x-2)$$

$$f'(x) = \frac{1}{x+2} + \frac{1}{x-2}$$

(c) (10 points) $f(x) = x^{3x}$ use logarithmic differentiation

$$\ln f(x) = 3x \cdot \ln x$$

$$\frac{1}{f(x)} \cdot f'(x) = 3\ln x + 3x \cdot \frac{1}{x}$$

$$f'(x) = (3\ln x + 3) \cdot x^{3x}$$

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2. (12 points) Find the equation for the tangent line at the point $(1, -1)$ of the curve given implicitly by:

$$x^2y - 3y^3 = x^2 + 1$$

$$2xy + x^2 \cdot y' - 9y^2 \cdot y' = 2x$$

@ $(1,-1)$: plug in $x=1$ & $y=-1$

$$2(1)(-1) + (1)^2 \cdot y' - 9 \cdot (-1)^2 y' = 2 \cdot (1)$$

$$-2 + y' - 9y' = 2$$

$$-8y' = 4$$

$$y' = -\frac{4}{8} = -\frac{1}{2}$$

tangent line:

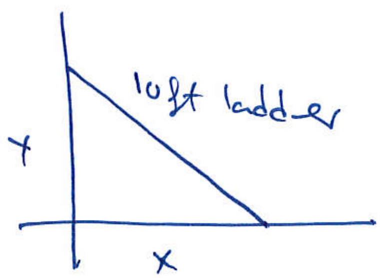
$$y + 1 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{1}{2} - 1$$

$$y = -\frac{1}{2}x - \frac{1}{2}$$

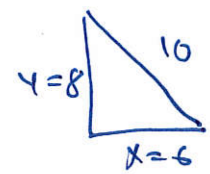
3. (12 points) A 10-foot-long ladder is resting against a vertical wall when Alice begins pulling the foot of the ladder away from the wall at a rate of 2 ft/sec.

(a) (8 points) How fast is the top of the ladder sliding down when the foot of the ladder is 6 ft from the wall?



Given: $\frac{dx}{dt} = 2$

Want: $\frac{dy}{dt}$ at the instant when



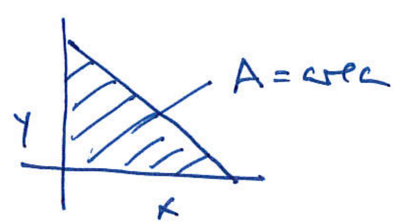
$$x^2 + y^2 = 10^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$

plug in $x=6$, $y=8$, $\frac{dx}{dt} = 2$

$$6 \cdot 2 + 8 \cdot \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{12}{8} = \boxed{-\frac{3}{2} \text{ ft/s}}$$

(b) (4 points) At what rate is the area of the triangle formed by the ladder, the wall, and the ground changing when the bottom of the ladder is 6 ft from the wall?



$A = \frac{1}{2}x \cdot y$

Want: $\frac{dA}{dt}$ at the instant when $x=6, y=8$

$A = \frac{1}{2}x \cdot y$ use product rule

Note $\frac{dy}{dt} = -\frac{3}{2}$ from part (a)

$\frac{dx}{dt} = 2$

$$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt} \cdot y + \frac{1}{2}x \cdot \frac{dy}{dt}$$

$$\frac{dA}{dt} = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot 8 + \frac{1}{2} \cdot 6 \cdot \left(-\frac{3}{2}\right) = \boxed{\frac{7}{2} \text{ ft}/s}$$

DO NOT WRITE ABOVE THIS LINE!!

4. (10 points) Find the maximum and minimum values of the function $f(x) = x^3 - 3x + 2$ on the interval $[-2, 0]$.

$f'(x) = 3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

use closed interval method

(i) $f' = 0 \Rightarrow x = -1$ ($x = +1$ is <u>not</u> in $[-2, 0]$)

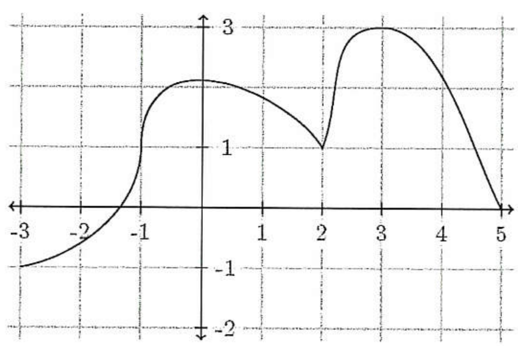
(ii) $f' = \emptyset$ N/A

(iii) endpoints $x = -2$ & $x = 0$

compare values:

x	$f(x)$
-2	0 = min value
-1	4 = max value
0	2

5. (8 points) The graph of the function $y = f(x)$ on the interval $[-3, 5]$ is given below. Note that this is the graph of the function f and NOT its derivative f'!



(a) (5 points) On what intervals is f increasing? decreasing? At what values of x does f have a local maximum? local minimum?

f is increasing on $(-3,0) \cup (2,3)$

decreasing on $(0,2) \cup (3,5)$

local max @ $x=0$ and @ $x=3$

local min @ $x=2$

Note that endpoints are <u>not</u> considered local extrema.

(b) (3 points) On what intervals is f concave upward? concave downward? Does f have any points of inflection?

f is concave up $(-3,-1)$

f is concave down $(-1,2) \cup (2,5)$

point of inflection @ $x=-1$

(Note that it appears that f might change concavity at $x \approx 4.9$ in which case we would have another point of inflection.)

6. (14 points) Consider the function $f(x) = (x+1)e^{-x}$, whose derivatives are given:

$$f'(x) = -xe^{-x}$$

$$f''(x) = (x-1)e^{-x}$$

(a) (8 points) On what intervals is f increasing? decreasing? At what values of x, if any, does f have a local maximum? local minimum?

sign of f' $\quad + \quad | \quad -$

$\nearrow \quad 0 \quad \searrow$

f increasing on $(-\infty, 0)$

decreasing on $(0, \infty)$

local max / global max @ $x=0$

no local min

(b) (6 points) On what intervals is f concave upward? concave downward? At what values of x, if any, does f have points of inflection?

sign of f'' $\quad - \quad | \quad +$

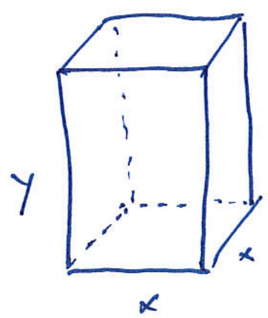
$\frown \quad 1 \quad \smile$

f is concave down $(-\infty, 1)$

concave up $(1, \infty)$

point of inflection @ $x=1$

7. (12 points) A fancy rectangular box with an open top and square base is to have a volume of 600 cm^3. Material for the base costs 12 cents per cm^2. The material for the sides is made out of different material and costs 10 cents per cm^2. Find the dimensions of the cheapest such box. Show all your work (this includes finding a domain for your optimization function), and make sure to justify your answer.



constraint:

$\text{Volume} = 600 = x^2 \cdot y$

$\Rightarrow y = \dfrac{600}{x^2}$

Minimize cost

$= \underbrace{12 \cdot x^2}_{\text{cost of botton}} + \underbrace{10(4xy)}_{\text{4 sides}}$ cents

$f(x) = 12x^2 + 40x \cdot \dfrac{600}{x^2}$

$= 12x^2 + \dfrac{24000}{x}$

Domain $= (0, \infty)$

$f'(x) = 24x - \dfrac{24000}{x^2} = 0$

$24x = \dfrac{24000}{x^2}$

$x^3 = \dfrac{24000}{24} = 1000$

$\Rightarrow x = 10$

sign of f'

$-$ $+$

0 $\searrow$ 10 $\nearrow$

absolute min when $x = 10$

& $y = \dfrac{600}{(10)^2} = 6$

	DIMENSIONS
height =	6 cm
width =	10 cm

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8. (10 points) Find the linear approximation of the function $f(x) = \arctan x$ at $a = 0$. Use this to approximate $\arctan(0.04)$ Write your final answer for $L(x)$ and your final approximation of $\arctan(0.04)$ in the box below.

$f(x) = \arctan x$

$f'(x) = \frac{1}{1+x^2}$

$f(0) = \arctan 0 = 0 \qquad f'(0) = \frac{1}{1+0} = 1$

$\Rightarrow L(x) = f(0) + f'(0)(x-0)$

$= 0 + 1\cdot(x-0)$

$\Rightarrow L(x) = x$

$\arctan(0.04) \approx 0.04$

$L(x) =$	x
$\arctan(0.04) \approx$	0.04