DO NOT WRITE ABOVE THIS LINE!!

MATH 181 Final Exam
December 14, 2017

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

- Circle your instructor: Jones Kashcheyeva Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- You must show all of your work.
- A solution for one problem may not go on another page.
- If you are asked to calculate an integral, make sure you justify your answer if it converges or diverges.

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1. (8 points) Evaluate $\int_0^{\pi/4} \cos^2 x \; dx$.

2. (8 points) Find $\int \sin^{-1} x \; dx$.

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3. (10 points) Use partial fractions to find $\displaystyle\int \frac{du}{1-u^2}$.

4. (10 points) Find $\displaystyle\int 5xe^{x^2}\ dx$.

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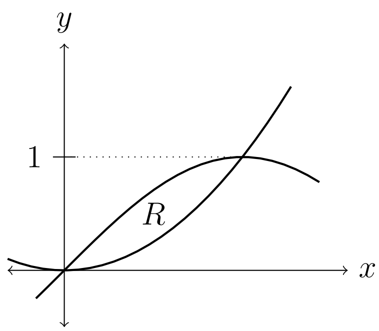
5. (12 points) Consider the region R bounded by $y = \sin x$ and $y = \dfrac{4}{\pi^2}x^2$, where both functions are graphed below.



(a) Find the area of R.

(b) Set up, BUT DO NOT EVALUATE, the integral representing the volume obtained by rotating the region R about the x-axis.

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6. (8 points) Find the slope of the tangent line to the parametric curve $x = t^2 - 4t$, $y = 3t^4$ at the point $t = 4$.

7. (8 points) A water tank has the shape of a cylinder with base radius 2 meters and height 6 meters. The tank is positioned vertically, so that its circular base is on the ground, and it has an outlet pipe at the top. It is half full of water. How much work is done to pump all of the water out of the outlet pipe? (Recall that the acceleration due to gravity is $g = 9.8$ meters per second per second and the mass density of water is $\rho = 1000$ kilograms per cubic meter.) Your answer should be in Newton-meters. Feel free to leave your answer in terms of g and ρ.

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8. (14 points) Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2n(x+1)^n}{3^n}$.

(a) State the center of the power series.

(b) Find the **radius** of convergence.

(c) Test the end points and state the interval of convergence.

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9. (12 points) (a) Find a power series representation for $f(x) = \sin(x^2)$. Either write your answer in sigma notation, or write down at least the first four non-zero terms.

(b) Use the first four non-zero terms of the series from part (a) to approximate the integral $\int_0^2 \sin(x^2)dx$. Do not simplify your approximation.

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10. (12 points) (a) Consider the power series

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad |x| < 1.$$

Using this power series, find a power series for $\ln(1-x)$. Use the sigma symbol below to write your final answer.

$$\ln(1-x) = \sum_{k=0}^{\infty}$$

(b) Now write your answer from part (a) with a change of index, starting at $n = 1$.

$$\ln(1-x) = \sum_{n=1}^{\infty}$$

(c) Using part (b), find the sum of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$.

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11. (10 points) Consider the matrix

$$A = \begin{bmatrix} -7 & -9 \\ 6 & 8 \end{bmatrix}$$

(a) Is the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ an eigenvector of A? If so, what is the corresponding eigenvalue?

(b) Given that -1 is an eigenvalue of A, find an eigenvector that corresponds to it.

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12. (12 points) (a) Convert the two rectangular (Cartesian) points to polar coordinates with $r > 0$ and $0 \le \theta < 2\pi$.

$(4, -4)$ and $(-1, \sqrt{3})$

(b) Convert the two polar points to rectangular (Cartesian) coordinates.

$\left(-3, \frac{\pi}{2}\right)$ and $\left(6, -\frac{5\pi}{4}\right)$

(c) Sketch the set of points $\left\{(r, \theta) : 1 \le r \le 4,\ \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}\right\}$.

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13. (12 points) Determine convergence or divergence of the following series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+10}$

(b) $\displaystyle\sum_{n=10}^{\infty} (-1)^n \cdot \frac{n^2}{\ln n}$

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14. (14 points) Consider the system

$$\begin{cases} 9x + 2y = -3 \\ x - 3y = -10 \end{cases}$$

(a) Write this system as an augmented matrix.

(b) Perform elementary operations to solve the system. Make sure you write down each of your elementary operations.

(c) Now take the same, original system and write it as a matrix equation $A\mathbf{x} = \mathbf{b}$ (by filling in the blanks below).

$$\underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -10 \end{bmatrix}$$

(d) Find $\det(A)$ and A^{-1}.

(e) Use part (d) to solve the system. [Note: Your answer here should match your answer to part (b).]