

DO NOT WRITE ABOVE THIS LINE!!

## MATH 181 Midterm 1

October 4, 2017

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

- Circle your instructor:                      Jones                      Kashcheyeva                      Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! ~~If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.~~
- You must show all of your work.
- A solution for one problem may not go on another page.
- If you are asked to calculate an integral, make sure you justify your answer if it converges or diverges.

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1. (10 points) Assume that  $g(x)$  is an even function, and also that  $\int_0^5 g(x) dx = 3$  and  $\int_5^7 g(x) dx = 4$ .

(a) Find  $\int_0^{-5} g(x) dx$ .

$$= \boxed{-3} \quad \text{since } \int_{-5}^0 g(x) dx = +3 \text{ using even symmetry}$$

(b) Find  $\int_{-5}^7 g(x) dx$ .

$$\begin{aligned} &= \int_{-5}^0 g(x) dx + \int_0^5 g(x) dx + \int_5^7 g(x) dx \\ &= \quad +3 \quad + 3 \quad + 4 \\ &= \boxed{10} \end{aligned}$$

- (c) Find the average value of  $g(x)$  on  $[5, 7]$ .

$$\text{ave value} = \frac{1}{7-5} \int_5^7 g(x) dx = \frac{1}{2} \cdot 4 = \boxed{2}$$

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2. (12 points) Calculate  $\int \cos^4 x \sin^3 x \, dx$

$$= \int \cos^4 x \sin^2 x \cdot \sin x \, dx$$

$$= \int \cos^4 x (1 - \cos^2 x) \sin x \, dx$$

u-sub

$$\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array}$$

$$= - \int u^4 (1 - u^2) \, du$$

$$= \int (-u^4 + u^6) \, du$$

$$= - \frac{u^5}{5} + \frac{u^7}{7} + C$$

$$= - \frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$$

3. (10 points) Set up, BUT DO NOT EVALUATE, an integral that represents the length along the curve  $f(x) = \ln(3x)$  from  $x = 1$  to  $x = 5$ .

$$= \int_1^5 \sqrt{1 + f'(x)} \, dx$$

$$f'(x) = \frac{3}{3x} = \frac{1}{x}$$

$$= \int_1^5 \sqrt{1 + \frac{1}{x^2}} \, dx$$

$$= \int_1^5 \sqrt{\frac{x^2 + 1}{x^2}} \, dx$$

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4. (12 points) Calculate  $\int x \tan^{-1} x \, dx$ . Note that  $\tan^{-1} x$  is the inverse tangent function.

[Hint:  $\frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2} = \frac{1+x^2}{1+x^2} - \frac{1}{1+x^2}$ .]

$$\boxed{\begin{array}{ll} u = \tan^{-1} x & dv = x \, dx \\ du = \frac{dx}{1+x^2} & v = \frac{1}{2}x^2 \end{array}}$$

by parts

$$= x^2 \tan^{-1} x - \int \frac{x^2}{1+x^2} \, dx \quad \text{using the hint}$$

$$= x^2 \tan^{-1} x - \int dx + \int \frac{1}{1+x^2} \, dx$$

$$= x^2 \tan^{-1} x - x + \frac{1}{2} \tan^{-1} x + c$$

5. (10 points) Calculate  $\int \frac{\sin(\ln x)}{x} \, dx$ .

$$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{dx}{x} \end{array}}$$

u-substitution

$$= \int \sin(u) \, du = -\cos(u) + c = -\cos(\ln x) + c$$

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6. (14 points)

(a) Find the partial fraction decomposition of  $\frac{1}{x^3 - x^2} = \frac{-1}{x} + \frac{-1}{x^2} + \frac{1}{x-1}$

$$x^3 - x^2 = x^2(x-1)$$

$$\frac{1}{x^3 - x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$1 = A(x-1) + B(x-1) + Cx^2$$

$$x=0 \quad 1 = 0 + B(-1) + 0 \Rightarrow B = -1$$

$$x=1 \quad 1 = 0 + 0 + C \Rightarrow C = +1$$

$$x=2 \quad 1 = A(2)(1) + B(2-1) + C(2^2)$$

$$1 = 2A + (-1) + 4$$

$$-2 = 2A$$

$$\Rightarrow A = -1$$

(b) Calculate  $\int \frac{1}{x^3 - x^2} dx$

$$= \int \frac{-1}{x} + \frac{-1}{x^2} + \frac{1}{x-1} dx$$

$$= -\ln|x| + \frac{1}{x} + \ln|x-1| + C$$

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7. (10 points) Consider the (improper) integral  $\int_2^4 \frac{dx}{(x-2)^2}$ .

(a) Write the improper integral in terms of a limit.

$$= \lim_{a \rightarrow 2^+} \int_a^4 \frac{dx}{(x-2)^2}$$

(b) Evaluate the integral, or show that the integral does not exist. State if the integral is convergent or divergent.

$$= \lim_{a \rightarrow 2^+} \left. -\frac{1}{x-2} \right|_a^4$$

$$= \lim_{a \rightarrow 2^+} \left( -\frac{1}{4-2} + \frac{1}{a-2} \right)$$

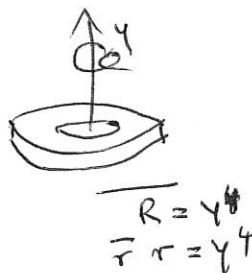
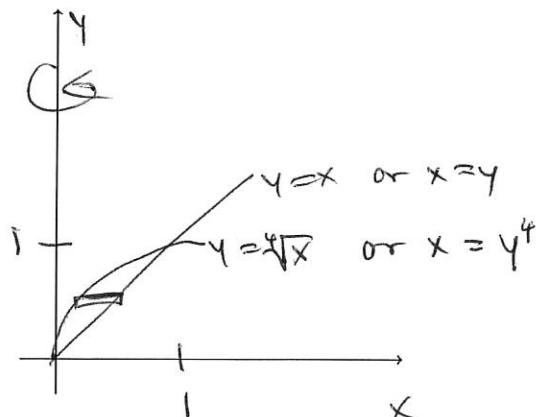
$$= \lim_{a \rightarrow 2^+} \left( -\frac{1}{2} + \frac{1}{a-2} \right) = +\infty$$

$\searrow$   $\nearrow$   
 $-\frac{1}{2}$   $+\infty$

diverges

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8. (10 points) Consider the region  $R$  in the first quadrant bounded by  $y = \sqrt[4]{x}$  and  $y = x$ . On the axes provided, sketch a graph of the region  $R$ , labelling the axes as well. Then find the volume obtained by rotating  $R$  about the  $y$ -axis.



$$\text{Volume} = \int_0^1 \pi \cdot ((y)^2 - (y^4)^2) dy$$

$$= \int_0^1 \pi (y^2 - y^8) dy$$

$$= \pi \left( \frac{y^3}{3} - \frac{y^9}{9} \right) \Big|_0^1$$

$$= \pi \left( \frac{1}{3} - \frac{1}{9} \right) - \pi (0 - 0)$$

$$= \pi \frac{3-1}{9}$$

$$= \boxed{\frac{2\pi}{9}}$$

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9. (12 points) Calculate  $\int_{-1}^{-\ln 2} 6te^{-t} dt$ . Simplify your answer. Your answer should not have any  $e$ 's in it.

by parts

$$\begin{array}{ll} u = 6t & dv = e^{-t} dt \\ du = 6 dt & v = -e^{-t} \end{array}$$

$$= -6te^{-t} \Big|_{-1}^{-\ln 2} + \int_{-1}^{-\ln 2} 6e^{-t} dt$$

$$= -6te^{-t} - 6e^{-t} \Big|_{-1}^{-\ln 2}$$

$$= \left( -6(-\ln 2)e^{-(-\ln 2)} - 6e^{-(-\ln 2)} \right) - \left( +6e^{-1} - 6e^{-1} \right)$$

$$= 6(\ln 2) \cdot 2 - 6 \cdot (2)$$

$$= 12 \ln 2 - 12$$