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MATH 181 Exam 2
October 31, 2018

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

- Circle your instructor's name: Simpson Diep Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- You must show all of your work.
- A solution for one problem may not go on another page.

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1. (10 points) Clearly circle your answers. If it is not clear which answer you circled, it will be marked incorrect.

(a) In the sequence $\{a_n\}$, if $\lim_{n\to\infty} a_n$ exists, then the sequence converges.

- TRUE
- FALSE

(b) In the series $\sum_k a_k$, if $\lim_{k\to\infty} a_k$ exists, then the series converges.

- TRUE
- FALSE

(c) When using the Ratio Test on the series $\sum_k a_k$, if $L = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = 0$,

then ________________________.

- the series converges
- the series diverges
- the Ratio Test is inconclusive

(d) Consider the power series $\sum_{k=2}^{\infty} \frac{3^k}{(2k)!}(x-4)^k$. What is the center of this series?

- -4
- -2
- 0
- 3
- 4

(e) Consider the series $-2+2-\frac{4}{3}+\frac{2}{3}+\cdots$. Choose the sigma notation below that yields this series.

- $\sum_{k=0}^{\infty} \frac{2^k}{k^2}$
- $\sum_{k=1}^{\infty} \frac{(-1)^k \cdot 2^k}{k!}$
- $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot k!}{2k+1}$
- $\sum_{k=0}^{\infty} \frac{(-1)^{k+1} \cdot 2 \cdot (2k)!}{k!}$

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2. (10 points) For each series below, determine if it converges or diverges, and state which test you used to form your conclusion.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{3}{4^k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} (2k+1)$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$

   (e) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3}$

3. (10 points) Consider the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = \dfrac{a_n + 3}{a_n + 1}$.

   (a) Write out the first three terms of this sequence.

   (b) Assuming that $\lim_{n\to\infty} a_n$ exists, find the limit of the sequence.

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4. (10 points) Consider the series

$$\sum_{k=0}^{\infty} \frac{2^{2k+1}}{5^{k-2}}.$$

(a) Write out the first three terms of the series.

(b) Determine whether the series converges or diverges. If it converges, find its sum and simplify your answer. Make sure you show your work.

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5. (10 points) Consider the series

$$\sum_{k=2}^{\infty} \frac{2}{k^2 - k}.$$

(a) Find the partial fraction decomposition of $\frac{2}{k^2 - k}$.

(b) Find a formula for S_n, the nth term in the sequence of partial sums. [Hint: You might want to write out the first few terms.]

(b) Using your formula for S_n determine if the series converges or diverges. If it converges, find what it converges to.

6. (10 points) Determine if the following series converges or diverges. $\sum_{k=1}^{\infty}(-1)^k|\sin(k)|$

7. (10 points) Determine if the following series converges or diverges. $\sum_{k=1}^{\infty}\frac{k^{100}}{k!}$

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8. (10 points) Determine if the following series converges or diverges. $\displaystyle\sum_{k=1}^{\infty} \frac{(\cos k)^2}{k^2}$

9. (10 points) Consider a function $f(x)$ such that $f(-1) = 4$, $f'(-1) = -1$, $f''(-1) = 2$, and $f'''(-1) = 21$.

   (a) Write the third order Taylor polynomial for $f(x)$ centered at $a = -1$.

   (b) Use your answer from (a) to estimate $f(0)$. Simplify your answer.

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10. (10 points) Find the interval of convergence of $\sum_{k=2}^{\infty} \frac{(-1)^k (x+4)^k}{k+1}$. Remember to test the endpoints.

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