DO NOT WRITE ABOVE THIS LINE!!

MATH 181 Exam 2
March 15, 2017

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ______________________________

University Email: ______________________________

UIN: ______________________________

Circle your instructor's name: Boester Cappetta Steenbergen

TA's Name: ______________________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (8 points) Given the sequence $a_n = \dfrac{12}{3n-1}$ starting at $n = 1$

   (a) Show that the sequence is monotonic.

   (b) Find a bound of the sequence.

   (c) True or False: From parts (a) and (b), the sequence must have a limit.

2. (8 points) Find a geometric series with starting term $a = 2$ that converges to a sum of 5.
   Write the series using $\sum$ notation or give the first four terms of the series. Hint: determine ratio r.
   DO NOT SIMPLIFY YOUR ANSWER.

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3. (16 points) Determine whether the following series converge or diverge. Justify your answer and name the test(s) used.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{7k+1}{2k^3-5k-3}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{2k^2-7k+9}{5k^2+4k+2}$

4. (20 points)

   (a) Does $\sum_{k=3}^{\infty} \frac{1}{\sqrt[3]{k-2}}$ converge? Justify your answer and name the test(s) used.

   (b) Does $\sum_{k=3}^{\infty} \frac{(-1)^k}{\sqrt[3]{k-2}}$ converge? Justify your answer and name the test(s) used.

   (c) Does $\sum_{k=3}^{\infty} \frac{(-1)^k}{\sqrt[3]{k-2}}$ converge absolutely or conditionally? Briefly explain why.

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5. (8 points) Find an expression for the n^{th} partial sum, S_n, of the series

$$\sum_{k=1}^{\infty} \left(\frac{1}{3k-2} - \frac{1}{3k+1} \right)$$

Does this series converge or diverge? Determine its value if it converges.

6. (8 points) For some function $f(x)$: $\quad f(3) = 1, \quad f'(3) = 2, \quad f''(3) = 4, \quad f'''(3) = 8.$

   (a) State the 3rd degree Taylor polynomial of $f(x)$.

   (b) Using part (a), approximate $f(3.1)$. DO NOT SIMPLIFY YOUR ANSWER.

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7. (18 points) For the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-3)^k}{k5^k}$

(a) Find $\displaystyle\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right|$.

(b) Use the result of (a) and the ratio test to find the radius of convergence.

(c) Test the endpoints using the result of (b) to find the interval of convergence.

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8. (14 points) Given the Maclaurin series: $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$

   (a) State the first 4 terms of the Maclaurin series for $2\tan^{-1}(x^2)$.

   (b) Using the terms in (a), approximate $\int_0^1 2\tan^{-1}(x^2)\, dx$. DO NOT SIMPLIFY YOUR ANSWER.