DO NOT WRITE ABOVE THIS LINE!!

MATH 210 Final Exam

December 14, 2017

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.

- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Lukina			Steenbergen	
Hong			Kobotis	
Dai			Bona	
Sinapova			Kashcheyeva	
Heard			Riedl	
Skalit			Dumas	
Nuer				

DO NOT WRITE ABOVE THIS LINE!!

1. **(10 pt)** Consider the three points $P(1,2,3)$, $Q(3,0,4)$ and $R(4,-2,3)$.

   (a) Compute $\overrightarrow{PQ} \cdot \overrightarrow{PR}$ and $\overrightarrow{PQ} \times \overrightarrow{PR}$.

   (b) Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Leave your answer in exact form.

   (c) Find the area of the triangle PQR.

2. **(10 pt)** Let C be the curve, given by the vector equation

$$\mathbf{r}(t) = \langle 2t, 3\cos t, 3\sin t\rangle, 0 \le t \le \pi.$$

(a) Find $\mathbf{r}'(t)$, $\mathbf{r}''(t)$ and $|\mathbf{r}'(t)|$.

(b) Find the unit tangent vector to the curve C at $t = \dfrac{\pi}{4}$.

DO NOT WRITE ABOVE THIS LINE!!

3. **(10 pt)** Consider the function

$$f(x, y) = 4x^2y - 4x^2 + 8x - 16y + 1.$$

Find all the critical points of f and classify them as local maxima, local minima or saddle points.

4. **(10pt)** Use the method of Lagrange multipliers to find the maximum and minimum values of

$$f(x, y, z) = x + 2y - z$$

subject to $x^2 + y^2 + z^2 = 1$.

DO NOT WRITE ABOVE THIS LINE!!

5. **(10 pt)** For the integral

$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dx\, dy$$

(a) Sketch the region of integration.

(b) Reverse the order of integration.

(c) Evaluate the integral from (b).

6. **(10pt)** Let C be a line segment from $(0,0)$ to $(2,2)$, and let $f(x,y) = x^2 + y$.

   (a) Write down a vector equation $\mathbf{r}(t)$ of the line segment, that is, find a parametrization of C.

   (b) Compute $\mathbf{r}'(t)$ and $|\mathbf{r}'(t)|$.

   (c) Compute the scalar line integral $\int_C f ds$.

7. **(10 pt)** Consider a conservative vector field $\mathbf{F} = \langle z^3 y, z^3 x + 2, 3z^2 xy + 1 \rangle$.

(a) Find a potential function for $\mathbf{F}$.

(b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a path from the point $A(0, 5, 0)$ to $B(5, 0, 1)$.

8. **(10pt)** Let D be the region enclosed by the circle $x^2 + y^2 = 4$ in the part of the plane where $x \geq 0$ and $y \geq 0$. Let C be the boundary curve of D equipped with counterclockwise orientation. Let $\mathbf{F} = \langle 2xy, x^2 \rangle$.

   (a) Sketch D and C (be sure to indicate the orientation of C).

   (b) Describe D in polar coordinates.

   (c) Using Green's Theorem, compute the outward flux $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$.

9. **(10 pt)** Given the vector field on $\mathbb{R}^3$,

$$\mathbf{F} = \langle 3xz, 2y^2 - y, 2x + 1 \rangle.$$

(a) Compute the divergence and the curl of $\mathbf{F}$.

(b) Determine if the vector field $\mathbf{F}$ is conservative. Justify your answer.

DO NOT WRITE ABOVE THIS LINE!!

10. **(10 pt)** Compute the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$, where S is the surface $z = \cos y$ for $0 \leq x \leq \frac{\pi}{2}$ and $0 \leq y \leq \frac{\pi}{2}$, oriented with the upward normal, and $\mathbf{F} = \langle 1, \cos x, y^2 \rangle$.