MATH 210 – Fall 2013
Final Exam
Thursday, December 12, 2013

Name (print) ___________________________ UIC ID ___________________________

By providing my signature, I pledge to abide by the University’s rules concerning academic honesty. This includes but is not limited to using unauthorized materials (cell phones, notes, books, calculators, etc.) or receiving/giving aid from/to another person.

Signature ________________________________________

Circle your instructor:
Cabrera Cheskidov Chou Dyer Gaster
Greenblatt Heard Hull Kashcheyeva
Kobotis Lowman Song Shvydkoy
Sparber Sward Terry

(1) Write your name, UIC ID, and signature in the spaces provided.
(2) Circle your instructor’s name and your discussion section time.
(3) There are 8 problems on this examination. Check to see that this copy is complete.
(4) All electronic devices are prohibited including calculators, cell phones, etc.
(5) Show your work. Answers without justification will receive little to no credit.
(6) If the space provided for your solution is not sufficient, complete your solution on the back of the previous page.

Do not write in this area.

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<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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SCORE _________/200
1. Let $\mathbf{F} = < 2y \cos(xy) + Dx, 2y + Dx \cos(xy)>$, where $D$ is a constant.

(a) Find a value for $D$ that makes $\mathbf{F}$ a conservative vector field.

(b) With $D$ as in (a), find a potential function for $\mathbf{F}$.

(c) With $D$ as in (a), evaluate the line integral of $\mathbf{F}$

$$\int_C \mathbf{F} \cdot \mathbf{T} \, ds,$$

where $C$ is the line segment from $(0, 0)$ to $(\sqrt{\pi}/2, \sqrt{\pi}/2)$. 
(30 pts) 2. Let $R$ be the region bounded by the parabola $y = x^2$ and line $y = x$. Let $C$ be the boundary of $R$ with counterclockwise orientation. Let $\mathbf{F} = (xy, x + y)$. Using Green’s theorem find the circulation of $\mathbf{F}$ along $C$. 
(20 pts) 3. (a) Find the point of intersection of the two lines \( \mathbf{r}(t) = (2 + 3t, 1 + t, 4 - t) \) and \( \mathbf{r}(t) = (4 + t, 2t, 2 + t) \).

(b) Find the equation of the plane containing the lines in part (a).
(30 pts) 4. Let \( R \) be the region in the first quadrant bounded by the curves \( y = \sqrt{9 - x^2} \), \( x = 0 \) and \( y = 0 \). Evaluate the double integral
\[
\int \int_R \frac{1}{e^{x^2+y^2}} \, dA.
\]
5. Let $C$ be the upper semicircle $\{(x, y) \mid x^2 + y^2 = 1, \ y \geq 0\}$, oriented in the counterclockwise direction. Evaluate the line integral

$$\int_C (xy + 1) \, dx + (x^2 + 2y) \, dy.$$
We will use Lagrange multipliers to find the maximum possible sum of the cosines of the angles of a Euclidean triangle: Let \( f(x, y, z) = \cos x + \cos y + \cos z \). Maximize \( f(x, y, z) \) subject to the constraints:

- \( x + y + z = \pi \)
- \( x \geq 0, y \geq 0, \text{ and } z \geq 0 \).
(20 pts) 7. Find the volume of the region between the surfaces $z = 10 - (x^2 + y^2)$ and $z = (x^2 + y^2) + 2$. 
(20 pts) 8. Let $S$ be the surface given by $f(x, y, z) = xz + y^2 = 0$.

(a) Find the equation of the tangent plane to $S$ at the point $P = (1, 2, -4)$.

(b) Find the parametric form of the line that passes through $P = (1, 2, -4)$ and is perpendicular to $S$ at that point.