First Hour Exam

(20 pts) 1. Do the following computations.
   (a) Compute \( \langle 1, 2, 3 \rangle \cdot \langle -2, 0, 1 \rangle \).
   (b) Compute \( \langle 1, -1, 3 \rangle \times \langle -2, -3, 1 \rangle \).
   (c) Find a normal vector to the plane described by \( 7x + 2y - 3z \).
   (d) Determine if the equations \( x - y + 2z = 1 \) and \( -x + y - 2z = 3 \) describe parallel planes, and give a reason.
   (e) If \( P = (4, 2, -3) \) and \( Q = (2, 1, 5) \), express the vector \( \overrightarrow{PQ} \) in terms of the standard unit vectors \( \mathbf{i}, \mathbf{j} \) and \( \mathbf{k} \).

(20 pts) 2. Consider the three points \( P = (2, -1, 3) \), \( Q = (2, 1, -2) \), and \( R = (1, 1, 0) \) in \( \mathbb{R}^3 \).
   (a) Find an equation for the plane which contains \( P, Q \) and \( R \).
   (b) Find the area of the triangle with vertices at \( P, Q \) and \( R \).

(15 pts) 3. Let \( c \) be the curve given by \( c(t) = \langle \cos 2t, 3t - 1, \sin 2t \rangle \).
   (a) Find parametric equations for the tangent line to \( c \) at \( t = \frac{\pi}{4} \).
   (b) Find the length of the curve \( c \) between \( t = -\pi \) and \( t = \pi \).
   (c) Find the curvature of \( c \) at \( t = 0 \).

(15 pts) 4. Find an equation for the tangent plane to the surface \( x^2 + 2y^2 - z^2 = 12 \) at the point \( (2, 2, 2) \).

(15 pts) 5. Find the linear function \( L(x, y) \) which gives the best linear approximation to the function \( f(x, y) = x \cos(\pi y) + ye^x \) at the point \( (1, 1) \).

(15 pts) 6. Let \( f(x, y) = \frac{x^2}{x^2 + y^2} \). Show that \( \lim_{(x,y) \to (0,0)} f(x, y) \) does not exist.

Hand in this sheet along with your exam booklet!