1. **(20pt)** For given vectors $\vec{u} = \langle -1, 1, \sqrt{6} \rangle$ and $\vec{v} = \langle 2, -2, 0 \rangle$ compute the following.

   (a) The sum $-2\vec{u} + \vec{v}$,

   (b) The dot product $\vec{u} \cdot \vec{v}$,

   (c) The angle between $\vec{u}$ and $\vec{v}$.

a) $-2\vec{u} + \vec{v} = \langle 2, -2, -2\sqrt{6} \rangle + \langle 2, -2, 0 \rangle =$

$\langle 4, -4, -2\sqrt{6} \rangle$

b) $\vec{u} \cdot \vec{v} = -2 - 2 + 0 = -4$

c) $|\vec{u}| = \sqrt{1+1+6} = \sqrt{8}$

$|\vec{v}| = \sqrt{4+4} = \sqrt{8}$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \frac{-4}{\sqrt{8}\sqrt{8}} = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

2. **(8 pt)** a) Find the equation of the line which is parallel to the line

$$\vec{r}(t) = \langle 3t, 1+4t, 2-5t \rangle$$

and which passes through the point $P(-1, 2, -2)$.

$\vec{v} = \langle 3, 4, -5 \rangle$

$\vec{r}(t) = \langle -1+3t, 2+4t, -2-5t \rangle$

(7 pt) b) Find the point (if it exists) at which the line $\vec{r}(t) = \langle 2t, 3-3t, -3+4t \rangle$ intersects the plane $z = 1$.

$1 = -3 + 4t$

$4t = 4$

$t = 1$

$\vec{r}(1) = \langle 2, 0, 1 \rangle,$

so the coordinates of the point of the intersection are $(2, 0, 1)$

3. **(15 pt)** A particle is moving in space with acceleration described by the function

$$\vec{a}(t) = \langle t, t^2, 1 \rangle.$$

At the moment $t = 0$ the instantaneous velocity of the particle is $\vec{v}_0 = \langle 1, 1, 1 \rangle$, and the coordinates of the particle are $\vec{r}_0 = \langle 2, 0, 0 \rangle$. Find the function $\vec{r}(t)$ describing the trajectory of the particle.

$$\vec{V}(t) = \left\langle \frac{t^2}{2}, \frac{t^3}{3}, t \right\rangle + \langle C_1, C_2, C_3 \rangle$$

$$\vec{V}(0) = \langle 0, 0, 0 \rangle + \langle C_1, C_2, C_3 \rangle = \langle 1, 1, 1 \rangle,$$

$$C_1 = 1, \quad C_2 = 1, \quad C_3 = 1$$

$$\vec{V}(t) = \left\langle \frac{t^2}{2} + 1, \frac{t^3}{3} + 1, t + 1 \right\rangle$$

$$\vec{r}(t) = \left\langle \frac{t^3}{6} + t, \frac{t^4}{12} + t, \frac{t^2}{2} + t \right\rangle + \langle D_1, D_2, D_3 \rangle$$

$$\vec{r}(0) = \langle D_1, D_2, D_3 \rangle = \langle 2, 0, 0 \rangle,$$

so $D_1 = 2, \quad D_2 = 0, \quad D_3 = 0.$

$$\vec{r}(t) = \left\langle \frac{t^3}{6} + t + 2, \frac{t^4}{12} + t, \frac{t^2}{2} + t \right\rangle$$

4. **(20 pt)** For the points $P(1,0,1)$, $Q(2,1,1)$ and $R(3,1,-1)$, find the following.

(a) The area of the parallelogram whose the sides are the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(b) The equation of the plane containing P, Q and R.

a) $\overrightarrow{PQ} = \langle 2-1, 1-0, 1-1 \rangle = \langle 1, 1, 0 \rangle$

$\overrightarrow{PR} = \langle 3-1, 1-0, -1-1 \rangle = \langle 2, 1, -2 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 2 & 1 & -2 \end{vmatrix} = \langle -2, 2, -1 \rangle$$

$$\text{area} = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{4+4+1} = 3$$

b) $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -2, 2, -1 \rangle$,

point $P(1,0,1)$

$$-2(x-1) + 2(y-0) - 1(z-1) = 0$$

$$-2x + 2y - z = -3$$

5. **(10 pt)** Find the length of a curve given by

$$\vec{r}(t) = \left\langle t, \frac{2}{3}(t-1)^{\frac{3}{2}} \right\rangle, \; 4 \le t \le 9.$$

$f(t) = t, \quad f'(t) = 1$

$g(t) = \frac{2}{3}(t-1)^{3/2} \quad g'(t) = (t-1)^{1/2}$

$L = \int_4^9 \sqrt{1+(t-1)}\, dt = \int_4^9 t^{1/2}\, dt =$

$\frac{2}{3} t^{3/2} \Big|_4^9 = \frac{2}{3}(3^3 - 2^3) =$

$\frac{38}{3}$

6. **(10 pt)** Find the equation of the line of the intersection of the planes $2x+y+z=1$ and $x-y-z=5$.

$\vec{n}_1 = \langle 2, 1, 1 \rangle$

$\vec{n}_2 = \langle 1, -1, -1 \rangle$

$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 0, 3, -3 \rangle$

point: set $z=0$ in both equations

$2x+y = 1$

$x-y = 5,$

solution $(x, y) = (2, -3)$, so the point in the intersection is $(2, -3, 0)$.

$\vec{r}(t) = \langle 2, -3+3t, -3t \rangle$