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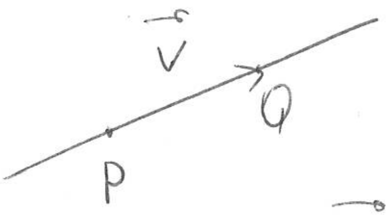
1. **(15pt)**

(a) Find an equation for a vector-valued function $\mathbf{r}(t)$ describing the line through the points $P(1,3,2)$ and $Q(-3,1,4)$.

(b) Find the coordinates of the point of the intersection of the line in a) and the plane $z = 4$



$\vec{v} = \overrightarrow{PQ} = \langle -3-1,\ 1-3,\ 4-2 \rangle =$
$\langle -4, -2, 2 \rangle$

$\vec{r}(t) = \langle 1, 3, 2 \rangle + t\langle -4, -2, 2 \rangle =$
$\langle 1-4t,\ 3-2t,\ 2+2t \rangle$

$2+2t = 4$
$2t = 2$
$t = 1$

$\vec{r}(1) = \langle 1-4, 3-2, 2+2 \rangle = \langle -3, 1, 4 \rangle$
$(-3, 1, 4)$

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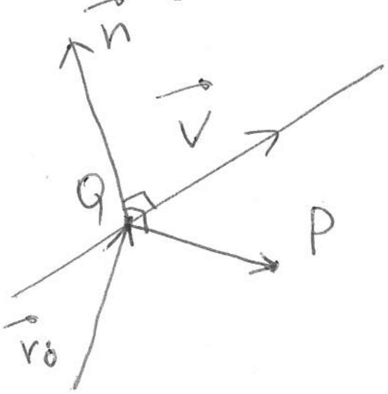
2. **(15pt)** Consider the line $\mathbf{r}(t) = \langle -2-5t, 3+2t, 4t\rangle$ and the point $P(2,-2,-1)$. Find an equation of the plane which contains the line $\mathbf{r}(t)$ and the point P.



$\vec{r}(0) = \langle -2, 3, 0\rangle, \quad Q(-2,3,0)$

$\overrightarrow{PQ} = \langle -2-2, 3-(-2), 0-(-1)\rangle = \langle -4, 5, 1\rangle$

$\vec{v} = \langle -5, 2, 4\rangle$

$$\vec{n} = \vec{v} \times \overrightarrow{PQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 2 & 4 \\ -4 & 5 & 1 \end{vmatrix} = \vec{i}(2-20) - \vec{j}(-5+16) + \vec{k}(-25+8) = \langle -18, -11, -17\rangle$$

$$-18(x-2) - 11(y+2) - 17(z+1) = 0$$

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3. **(15 pt)** A particle moves in space with acceleration given by the vector-valued function

$$\mathbf{a}(t) = \langle 9 \sin 3t, 6t^2, 9 \cos 3t \rangle.$$

Time t is measured in seconds. At time $t = 0$ the particle is stationary, that is, $\mathbf{v} = \langle 0, 0, 0 \rangle$, and located at the point with coordinates $(2, -5, 3)$.

(a) Compute the velocity function $\mathbf{v}(t)$.

(b) Compute the position function $\mathbf{r}(t)$.

a) $\vec{v}(t) = \int \vec{a}(t)dt = \langle -\frac{9}{3} \cos 3t, 6\frac{t^3}{3}, \frac{9}{3} \sin 3t \rangle + \langle C_1, C_2, C_3 \rangle = \langle -3 \cos 3t, 2t^3, 3 \sin 3t \rangle + \langle C_1, C_2, C_3 \rangle$

$\vec{v}(0) = \langle -3 \cos 0, 0, 0 \rangle + \langle +C_1, C_2, C_3 \rangle =$

$\langle -3, 0, 0 \rangle + \langle C_1, C_2, C_3 \rangle = \langle 0, 0, 0 \rangle$

$C_1 - 3 = 0 \quad C_2 = 0 \quad C_3 = 0$

$C_1 = 3$

$\vec{v}(t) = \langle -3 \cos 3t + 3, 2t^3, 3 \sin 3t \rangle$

$\vec{r}(t) = \int \vec{v}(t)dt = \langle -3\frac{1}{3} \sin 3t + 3t, 2\frac{t^4}{4},$

$-\frac{3}{3} \cos 3t \rangle + \langle D_1, D_2, D_3 \rangle =$

$\langle -\sin 3t + 3t, \frac{1}{2}t^4, -\cos 3t \rangle + \langle D_1, D_2, D_3 \rangle$

$\vec{r}(0) = \langle D_1, D_2, -1 + D_3 \rangle = \langle 2, -5, 3 \rangle$

$D_1 = 2, \quad D_2 = -5 \quad -1 + D_3 = 3$

$D_3 = 4$

$\vec{r}(t) = \langle -\sin 3t + 3t + 2, \frac{1}{2}t^4 - 5, -\cos 3t + 4 \rangle$

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4. **(10 pt)** Let $z = f(x, y) = \sqrt{x^2 - 1 - y}$.

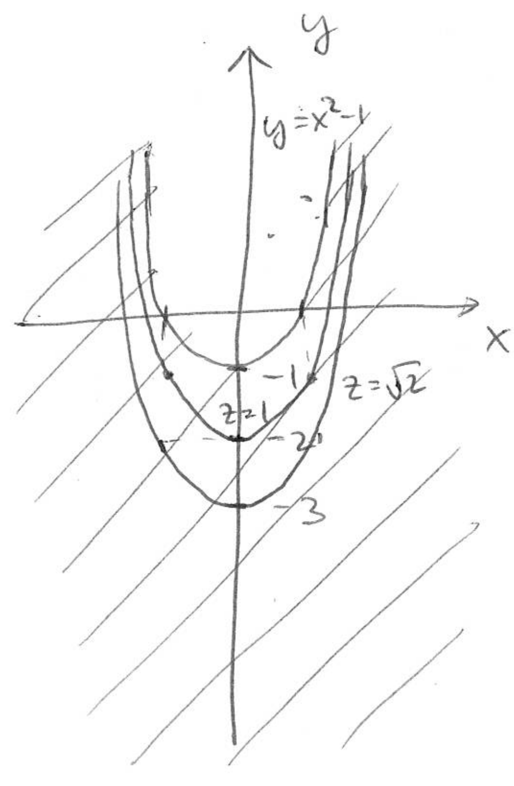
(a) State the domain of f. Give a rough sketch of this region in the xy plane; be sure to shade in the area belonging to the domain.

(b) Find the equations of level curves for $z = 1$ and for $z = \sqrt{2}$. Sketch the level curves on the same picture as the domain of f.

a) $D = \{(x,y) \mid x^2 - 1 - y \geq 0\}$

$x^2 - 1 - y \geq 0$

$y \leq x^2 - 1$



b)

$1 = \sqrt{x^2 - 1 - y}$

$1 = x^2 - 1 - y$

$y = x^2 - 2$

$\sqrt{2} = \sqrt{x^2 - 1 - y}$

$2 = x^2 - 1 - y$

$y = x^2 - 3$

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5. **(10 pt)** Let $\mathbf{r}(t) = \langle t^2 - 1,\ 3t^2 - 6,\ t^2 + 2t \rangle$.

(a) Compute $\mathbf{r}'(t)$.

(b) Compute the tangent vector for $t = 1$.

(c) Compute $\int_0^1 \mathbf{r}(t)\, dt$.

a) $\vec{r}\,'(t) = \langle 2t, 6t, 2t+2 \rangle$

b) $\vec{r}\,'(1) = \langle 2, 6, 4 \rangle$

c) $\int_0^1 \vec{r}(t)\,dt = \langle \left(\frac{t^3}{3} - t\right)\Big|_0^1, \left(3\frac{t^3}{3} - 6t\right)\Big|_0^1,$

$\left(\frac{t^3}{3} + 2\frac{t^2}{2}\right)\Big|_0^1 \rangle =$

$\langle \frac{1}{3} - 1,\ 1 - 6,\ \frac{1}{3} + 1 \rangle = \langle -\frac{2}{3}, -5, \frac{4}{3} \rangle$

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6. **(10 pt)** A surface is given by the equation

$$z = \sqrt{x^2 + y^2}.$$

(a) Find the equations of the xy-, yz- and xz-traces of the surface.

(b) Find the equation of the trace in the plane $z = 1$.

(c) Sketch the surface using the traces you found in a) and b).

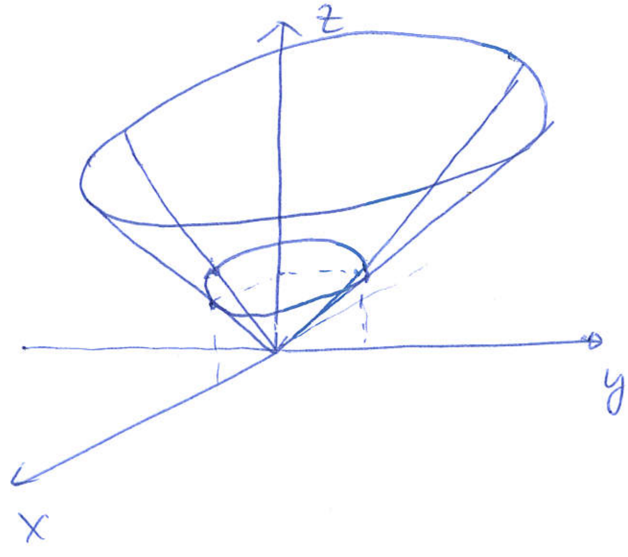
a) xy-trace: $z=0$ $\sqrt{x^2+y^2}=0$, $(x,y)=(0,0)$ a point.

yz-trace: $x=0$, $z=\sqrt{y^2}$,
$z=y$ if $y\geq 0$, $z=-y$ if $y<0$

xz-trace: $y=0$, $z=\sqrt{x^2}$,
$z=x$ if $x\geq 0$, $z=-x$ if $x<0$.

b) $1=\sqrt{x^2+y^2}$
$x^2+y^2=1$

c)



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7. **(10 pt)** Find all four second order partial derivatives of the function

$$f(x,y) = x(x-y^2) = x^2 - xy^2$$

and evaluate them at the point $(2,-1)$.

$f_x = 2x - y^2$ $\qquad$ $f_y = -2xy$

$f_{xx} = 2$ $\qquad$ $f_{yx} = -2y$

$f_{xy} = -2y$ $\qquad$ $f_{yy} = -2x$

$f_{xx}(2,-1) = 2$

$f_{xy}(2,-1) = f_{yx}(2,-1) = -2\cdot(-1) = 2$

$f_{yy}(2,-1) = -2\cdot 2 = -4$

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8. **(15 pt)** Consider the function $f(x,y) = x + xy + y^2$.

(a) Compute the gradient $\nabla f(x,y)$.

(b) Find the directional derivative $D_{\mathbf{u}}f(1,1)$, where $\mathbf{u} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.

(c) Compute the unit vector in the direction of maximum increase at $(1,1)$.

(d) What is the rate of maximum increase in c)?

a) $f_x = y + 1 \qquad f_y = x + 2y$

$\nabla f(x,y) = \langle y+1, x+2y \rangle$

b) $D_u f(1,1) = \langle 1+1, 1+2 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle =$

$\frac{2}{\sqrt{2}} + 3/\sqrt{2} = \frac{5}{\sqrt{2}}$

c) $\nabla f(1,1) = \langle 2, 3 \rangle . \qquad |\nabla f(1,1)| = \sqrt{4+9} = \sqrt{13}$

$\frac{\nabla f(1,1)}{|\nabla f(1,1)|} = \frac{\langle 2,3 \rangle}{\sqrt{13}} = \langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \rangle$

d) $|\nabla f(1,1)| = \sqrt{13}$