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MATH 210 Exam 1

October 4, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded!

- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Abramov		Hachtman		Kobotis	
Dai		Hamdan		Lukina	
Datta		Heard		Pourarian	
Freitag		Jones		Rosendal	
Greenblatt		Kashcheyeva		Sparber	
				Townsend	

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1. **(15pt)** Consider two vectors $\mathbf{u} = \langle 1, 0, 2 \rangle$ and $\mathbf{v} = \langle 0, 1, -1 \rangle$.

   (a) Compute $3\mathbf{u} - 2\mathbf{v}$.

   (b) Find the dot product $\mathbf{u} \cdot \mathbf{v}$.

   (c) Find the cross product $\mathbf{u} \times \mathbf{v}$.

   (d) Find the area of the parallelogram which has the vectors $\mathbf{u}$ and $\mathbf{v}$ as sides.

2. **(10pt)** Consider the plane $2x - 3y + 6z = 1$.

   (a) Find a vector normal to the plane.

   (b) Find a vector equation of the line which is orthogonal to the plane and passes through the point $(1, 0, -2)$.

3. **(15pt)** Consider the vector-valued function.

$$\mathbf{r}(t) = \langle 10\cos t, 5\sin t\rangle$$

(a) Find the derivative $\mathbf{r}'(t)$.

(b) Find the vectors $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ at $t = \pi/4$.

(c) Find the angle between the vectors in (b). Leave your answer in exact form.

4. **(10 pt)** Suppose an object has velocity given by $\mathbf{v}(t) = \langle 2t, t^2, e^{2t} \rangle$.

   (a) Find the object's position function $\mathbf{r}(t)$ if $\mathbf{r}(0) = \langle 0, 0, 0 \rangle$.

   (b) Find the object's speed at the moment of time $t = 1$.

   (c) Find the object's acceleration $\mathbf{a}(t)$.

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5. **(15 pt)** Find an equation for the plane containing the points $A(1,1,1)$, $B(1,4,3)$ and $C(-1,2,1)$.

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6. **(10 pt)** Find the length of the curve parametrised by

$$\mathbf{r}(t) = 5\cos(t^2)\mathbf{i} + 4\sin(t^2)\mathbf{j} + 3\sin(t^2)\mathbf{k}, \quad 0 \leqslant t \leqslant 2.$$

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7. **(10 pt)** Suppose $z = x^2y + e^x$, where $x = \cos(s - t)$ and $y = st^2$.

   Use the Chain Rule to compute $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

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8. **(15 pt)** Consider the function

$$f(x, y, z) = e^{x-y} + z^2(x + y).$$

(a) Compute the partial derivatives of f at $P(2, 2, 1)$.

(b) Compute the vector $\overrightarrow{PO}$, where $O(0, 0, 0)$, and find the unit vector that has the same direction as $\overrightarrow{PO}$.

(c) Compute the directional derivative of f at $P(2, 2, 1)$ in the direction of the vector $\overrightarrow{PO}$.