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MATH 210 Exam 1

October 4, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded!

First name (please write as legibly as possible within the boxes)

Last name

UIC NetID (the part of your email address before @uic.edu)

- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Abramov		Hachtman		Kobotis	
Dai		Hamdan		Lukina	
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1. **(15pt)** Consider two vectors $\mathbf{u} = \langle 1, 0, 2 \rangle$ and $\mathbf{v} = \langle 0, 1, -1 \rangle$.

   (a) Compute $3\mathbf{u} - 2\mathbf{v}$.

   (b) Find the dot product $\mathbf{u} \cdot \mathbf{v}$.

   (c) Find the cross product $\mathbf{u} \times \mathbf{v}$.

   (d) Find the area of the parallelogram which has the vectors $\mathbf{u}$ and $\mathbf{v}$ as sides.

a) $3\vec{u} - 2\vec{v} = 3\langle 1,0,2\rangle - 2\langle 0,1,-1\rangle =$

$\langle 3-0, 0-2, 6+2\rangle = \langle 3, -2, 8\rangle$

b) $\vec{u} \cdot \vec{v} = \langle 1,0,2\rangle \cdot \langle 0,1,-1\rangle = -2$

c) $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{vmatrix} = \vec{i}(0-2) - \vec{j}(-1-0) + \vec{k}(1-0) =$

$\langle -2, 1, 1\rangle$

d) area $= |\vec{u} \times \vec{v}| = \sqrt{4+1+1} = \sqrt{6}$

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2. **(10pt)** Consider the plane $2x - 3y + 6z = 1$.

   (a) Find a vector normal to the plane.

   (b) Find a vector equation of the line which is orthogonal to the plane and passes through the point $(1, 0, -2)$.

a) $\vec{n} = \langle 2, -3, 6 \rangle$

b) $\vec{r}(t) = \langle 1, 0, -2 \rangle + t \langle 2, -3, 6 \rangle$

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3. **(15pt)** Consider the vector-valued function.

$$\mathbf{r}(t) = \langle 10\cos t, 5\sin t\rangle$$

(a) Find the derivative $\mathbf{r}'(t)$.

(b) Find the vectors $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ at $t = \pi/4$.

(c) Find the angle between the vectors in (b). Leave your answer in exact form.

a) $\vec{r}'(t) = \langle -10\sin t, 5\cos t\rangle$

b) $\vec{r}\left(\frac{\pi}{4}\right) = \langle 10\cos\frac{\pi}{4}, 5\sin\frac{\pi}{4}\rangle = \langle \frac{10\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\rangle$

$\vec{r}'\left(\frac{\pi}{4}\right) = \langle -10\sin\frac{\pi}{4}, 5\cos\frac{\pi}{4}\rangle =$
$\langle -\frac{10\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\rangle$

c) $\vec{r}\left(\frac{\pi}{4}\right)\cdot\vec{r}'\left(\frac{\pi}{4}\right) = \langle 5\sqrt{2}, \frac{5\sqrt{2}}{2}\rangle\cdot\langle -5\sqrt{2}, \frac{5\sqrt{2}}{2}\rangle =$
$-25\cdot 2 + \frac{25\cdot 2}{4} = -50 + \frac{25}{2} =$
$-\frac{75}{2}$

$|\vec{r}'\left(\frac{\pi}{4}\right)| = \sqrt{\frac{100\cdot 2}{4} + \frac{25\cdot 2}{4}} = \sqrt{\frac{125}{2}}$

$|\vec{r}\left(\frac{\pi}{4}\right)| = \sqrt{\frac{100\cdot 2}{4} + \frac{25\cdot 2}{4}} = \sqrt{\frac{125}{2}}$

$\cos\theta = \frac{-75/2}{\frac{125}{2}} = -\frac{75}{125} = -\frac{3}{5}$

$\theta = \cos^{-1}\left(-\frac{3}{5}\right)$

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4. **(10 pt)** Suppose an object has velocity given by $\mathbf{v}(t) = \langle 2t, t^2, e^{2t} \rangle$.

   (a) Find the object's position function $\mathbf{r}(t)$ if $\mathbf{r}(0) = \langle 0, 0, 0 \rangle$.

   (b) Find the object's speed at the moment of time $t = 1$.

   (c) Find the object's acceleration $\mathbf{a}(t)$.

a) $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 2\frac{t^2}{2} + C_1, \frac{t^3}{3} + C_2, \frac{1}{2}e^{2t} + C_3 \rangle$

$\vec{r}(0) = \langle C_1, C_2, \frac{1}{2} + C_3 \rangle = \langle 0, 0, 0 \rangle$

$C_1 = 0, \quad C_2 = 0, \quad C_3 = -\frac{1}{2}$

$\vec{r}(t) = \langle t^2, \frac{t^3}{3}, \frac{1}{2}e^{2t} - \frac{1}{2} \rangle$

b) $\vec{v}(1) = \langle 2, 1, e^2 \rangle$

$|\vec{v}(1)| = \sqrt{4 + 1 + e^4} = \sqrt{5 + e^4}$

c) $\vec{a}(t) = \vec{v}'(t) = \langle 2, 2t, 2e^{2t} \rangle$

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5. **(15 pt)** Find an equation for the plane containing the points $A(1,1,1)$, $B(1,4,3)$ and $C(-1,2,1)$.

$\vec{AB} = \langle 1-1, 4-1, 3-1 \rangle = \langle 0, 3, 2 \rangle$

$\vec{AC} = \langle -1-1, 2-1, 1-1 \rangle = \langle -2, 1, 0 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 2 \\ -2 & 1 & 0 \end{vmatrix} =$$

$$\vec{i}(0-2) - \vec{j}(0+4) + \vec{k}(0+6) = \langle -2, -4, 6 \rangle$$

$$\boxed{-2(x-1) - 4(y-1) + 6(z-1) = 0}$$

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6. **(10 pt)** Find the length of the curve parametrised by

$$\mathbf{r}(t) = 5\cos(t^2)\mathbf{i} + 4\sin(t^2)\mathbf{j} + 3\sin(t^2)\mathbf{k}, \quad 0 \leqslant t \leqslant 2.$$

$f(t) = 5 \cos t^2 \qquad f'(t) = -2t \cdot 5 \sin t^2$

$g(t) = 4 \sin t^2 \qquad g'(t) = +2t \cdot 4 \cos t^2$

$h(t) = 3 \sin(t^2) \qquad h'(t) = 2t \cdot 3 \sin t^2$

$\vec{r}'(t) = \langle -10t \sin t^2, 8t \cos t^2, 6t \sin t^2 \rangle$

$|\vec{r}'(t)| = \sqrt{100t^2 \sin^2 t^2 + 64t^2 \cos^2 t^2 + 36t^2 \sin^2 t^2}$

$\sqrt{100t^2} = 10t$

$L = \int_0^2 10t \, dt = 10 \frac{t^2}{2} \Big|_0^2 = 5 \cdot 4 = 20$

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7. **(10 pt)** Suppose $z = x^2y + e^x$, where $x = \cos(s-t)$ and $y = st^2$.

Use the Chain Rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

$z_x = 2xy + e^x$ $\qquad z_y = x^2$

$\frac{\partial x}{\partial t} = -(-\sin(s-t)) = \sin(s-t)$

$\frac{\partial x}{\partial s} = -\sin(s-t)$

$\frac{\partial y}{\partial s} = t^2$ $\qquad \frac{\partial y}{\partial t} = 2st$

$\frac{\partial z}{\partial s} = (2xy + e^x)(-\sin(s-t)) + x^2 t^2 =$

$-(2\cos(s-t)st^2 + e^{\cos(s-t)})\sin(s-t) + t^2\cos^2(s-t)$

$\frac{\partial z}{\partial t} = (2xy + e^x)\sin(s-t) + x^2 2st =$

$((2\cos(s-t)st^2 + e^{\cos(s-t)})\sin(s-t) +$

$2st\cos^2(s-t)$

8. **(15 pt)** Consider the function

$$f(x,y,z) = e^{x-y} + z^2(x+y).$$

(a) Compute the partial derivatives of f at $P(2,2,1)$.

(b) Compute the vector $\overrightarrow{PO}$, where $O(0,0,0)$, and find the unit vector that has the same direction as $\overrightarrow{PO}$.

(c) Compute the directional derivative of f at $P(2,2,1)$ in the direction of the vector $\overrightarrow{PO}$.

a) $f_x = e^{x-y} + z^2$ $\quad f_y = -e^{x-y} + z^2$ $\quad f_z = (x+y)2z$

$f_x(2,2,1) = e^{2-2} + 1 = 2$ $\quad f_y = -e^{2-2} + 1 = 0$

$f_z = (2+2)\cdot 2 \cdot 1 = 8$

$\nabla f(2,2,1) = \langle 2, 0, 8 \rangle$

b) $\overrightarrow{PO} = \langle -2, -2, -1 \rangle$ $\quad |\overrightarrow{PQ}| = \sqrt{4+4+1} = 3$

$\vec{u} = \dfrac{\overrightarrow{PO}}{|\overrightarrow{PO}|} = \langle -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3} \rangle$

c) $D_{\vec{u}} f(2,2,1) = \nabla f(2,2,1) \cdot \vec{u} =$

$\langle 2, 0, 8 \rangle \cdot \langle -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3} \rangle =$

$-\frac{4}{3} - \frac{8}{3} = -\frac{12}{3} = -4$