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MATH 210 Exam 1

February 16, 2016

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

Name: _____

UIN: _____

University Email: _____

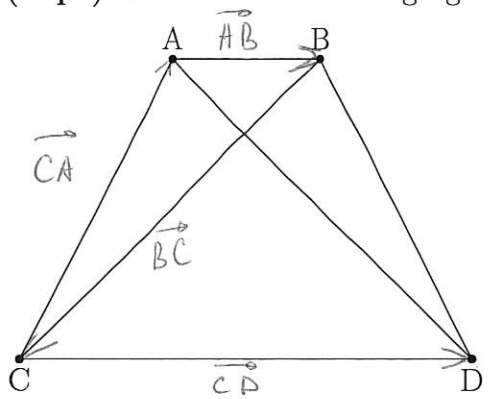
Check next to your instructor's name:

Lukina	12pm	
Lukina	2pm	
Kobotis	8am	
Whyte	10am	
Lenz	11am	
Abernethy	12pm	
Heard	9am	
Wang	2pm	
Kauffman	3pm	

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. **(10pt)** Consider the following figure:



Write each sum or difference as a single vector in terms of A, B, C, and D.

(a) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(b) $\overrightarrow{CA} - \overrightarrow{CD} = \overrightarrow{DA}$

(c) $\overrightarrow{DA} + \overrightarrow{CB} + \overrightarrow{AC} = \overrightarrow{DB}$

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2. **(10 pt)** Find the equation of the line which is parallel to the line

$$\vec{r}(t) = \langle 1 - t, 1 + t, 2 \rangle$$

and which passes through the point $P(0, 1, -2)$.

$\vec{v} = \langle -1, 1, 0 \rangle$

$\vec{r}(t) = \langle 0, 1, -2 \rangle + t\langle -1, 1, 0 \rangle =$

$\langle -t, 1+t, -2 \rangle$

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3. **(15pt)** Given vectors $\vec{u} = \langle -2, 1, 1 \rangle$ and $\vec{v} = \langle -1, 0, 1 \rangle$,

   (a) Compute the dot product $\mathbf{u} \cdot \mathbf{v}$,

   (b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

a) $\vec{u} \cdot \vec{v} = \langle -2, 1, 1 \rangle \cdot \langle -1, 0, 1 \rangle = 2 + 1 = 3$

b) $|\vec{u}| = \sqrt{4 + 1 + 1} = \sqrt{6}$

$|\vec{v}| = \sqrt{1 + 1} = \sqrt{2}$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \frac{3}{\sqrt{2 \cdot 6}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

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4. **(15 pt)** A particle is moving in space with acceleration described by the function

$$\mathbf{a}(t) = \langle 1, \cos t, \sin t \rangle.$$

At time $t = 0$ the particle is stationary and it is located at the origin $(0, 0, 0)$.

(a) Compute the position function $\mathbf{r}(t)$.

(b) Write down an integral that will give the total distance traveled by the particle during the first second of its travel. Do not evaluate.

a) $\vec{v}(t) = \int \vec{a}(t)\,dt = \langle t, \sin t, -\cos t \rangle + \langle C_1, C_2, C_3 \rangle$

$\vec{v}(0) = \langle 0, \sin 0, -\cos 0 \rangle + \langle C_1, C_2, C_3 \rangle = \langle 0, 0, 0 \rangle$

$\langle C_1, C_2, -1 + C_3 \rangle = \langle 0, 0, 0 \rangle$, so

$C_1 = 0, \quad C_2 = 0, \quad C_3 = 1, \quad \vec{v}(t) = \langle t, \sin t, -\cos t + 1 \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{t^2}{2}, -\cos t, -\sin t + t \rangle + \langle D_1, D_2, D_3 \rangle$

$\vec{r}(0) = \langle 0, -1, 0 \rangle + \langle D_1, D_2, D_3 \rangle = \langle 0, 0, 0 \rangle,$

so $D_1 = 0, \quad D_2 = 1, \quad D_3 = 0.$

$\vec{r}(t) = \langle \frac{t^2}{2}, -\cos t + 1, -\sin t + t \rangle$

b) $\vec{v}(t) = \vec{r}\,'(t) = \langle t, \sin t, -\cos t + 1 \rangle,$

so

$$L = \int_0^1 \sqrt{t^2 + \sin^2 t + (1 - \cos t)^2}\; dt$$

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5. **(15 pt)** Give an equation of the plane that contains the points $O(0,0,0)$ (the origin), $P(1,2,3)$ and $Q(1,1,1)$.

$\overrightarrow{OP} = \langle 1, 2, 3 \rangle$, $\overrightarrow{OQ} = \langle 1, 1, 1 \rangle$,

$$\vec{n} = \overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \vec{i}(2-3) - \vec{j}(1-3) + \vec{k}(1-2),$$

so $\vec{n} = \langle -1, 2, -1 \rangle$

point $O(0,0,0)$: $-1(x-0) + 2(y-0) - 1(z-0) = 0$

$$-x + 2y - z = 0$$

or; using $P(1,2,3)$:

$$-1(x-1) + 2(y-2) - 1(z-3) = 0$$

$$-x + 2y - z = 0$$

or, using $Q(1,1,1)$:

$$-1(x-1) + 2(x-1) - 1(x-1) = 0$$

$$-x + 2y - z = 0.$$

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6. **(10 pt)** Are the planes given by the equations

$$2x - y + z = 1 \quad \text{and} \quad 2x + 3y - z = 2$$

orthogonal? Justify your answer (yes/no answer without justification will be given no credit).

$\vec{n}_1 = \langle 2, -1, 1 \rangle$, $\vec{n}_2 = \langle 2, 3, -1 \rangle$

$\vec{n}_1 \cdot \vec{n}_2 = \langle 2, -1, 1 \rangle \cdot \langle 2, 3, -1 \rangle =$

$4 - 3 - 1 = 0$,

yes, the planes are orthogonal.

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7. **(15 pt)** A surface is given by the equation

$$\frac{x^2}{4} + \frac{y^2}{9} - z = 0.$$

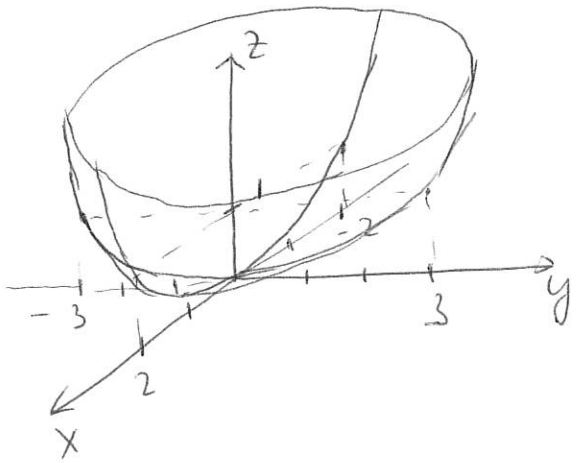
(a) Find the equations of the xy-, yz- and xz-traces of the surface.

(b) Sketch the surface.

xy-trace: $z = 0$, $\frac{x^2}{4} + \frac{y^2}{9} = 0$,
the only solution $x = y = 0$,
the trace is a point $(0,0,0)$.

yz-trace: $x = 0$, $\frac{y^2}{9} - z = 0$
$z = \frac{y^2}{9}$ - parabola

xz-trace: $y = 0$, $z = \frac{x^2}{4}$ - parabola



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8. **(10 pt)** Given a function of two variables

$$f(x,y) = \sqrt{x^2+y^2-1},$$

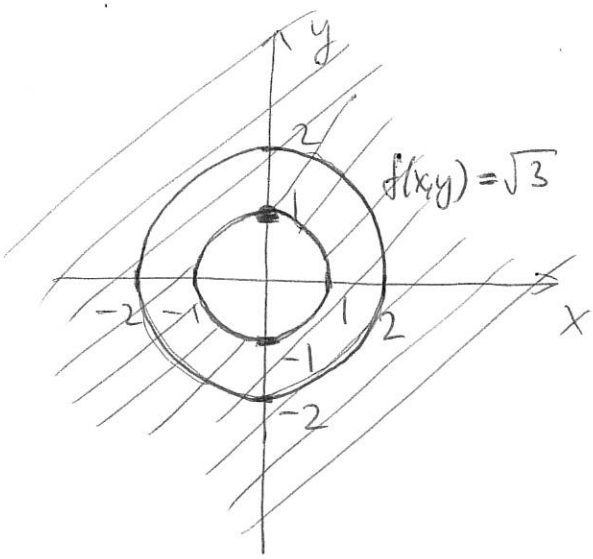
(a) Find and sketch its domain (use a solid curve to indicate that the domain includes the boundary, and a dashed curve to indicate that the domain excludes the boundary).

(b) Sketch the level curve $f(x,y) = \sqrt{3}$ on the same coordinate grid.

a) $x^2+y^2-1 \geq 0$

$x^2+y^2 \geq 1$,

domain $\{(x,y) \mid x^2+y^2 \geq 1\}$



b) $\sqrt{3} = \sqrt{x^2+y^2-1}$

$x^2+y^2-1 = 3$

$x^2+y^2 = 4$

circle of radius 2