DO NOT WRITE ABOVE THIS LINE!!

MATH 210 Exam 2

March 16, 2017

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

Name: _____

UIN: _____

University Email: _____

Check next to your instructor's name:

Lukina		
Abramov		
Heard		
Woolf		
Thulin		
Page		
Skalit		
Kobotis		
Freitag		
Shulman		
Lesieutre		

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

DO NOT WRITE ABOVE THIS LINE!!

1. **(10 pt)** Find an equation of the tangent plane to the sphere

$$x^2 + y^2 + z^2 = 1$$

at the point $(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{2})$.

DO NOT WRITE ABOVE THIS LINE!!

2. **(10 pt)** Consider the function

$$f(x, y) = x^2 + 4y^2.$$

(a) Find the equation of the plane tangent to the graph of the function at the point $(2, 1, 8)$.

(b) Use a linear approximation to the function $f(x, y)$ to estimate $f(1.9, 1.1)$. Your answer should be a single number in decimal form, or written as a reduced fraction.

DO NOT WRITE ABOVE THIS LINE!!

3. **(15 pt)** Find all critical points of the function

$$f(x, y) = x^3 + 2xy + y^2 - x$$

and classify them using the Second Derivative Test.

DO NOT WRITE ABOVE THIS LINE!!

4. **(15 pt)** Let R be the region enclosed by the line $y = 4$ and the parabola $y = x^2$. Find the absolute maximum and minimum values attained by $f(x, y) = 2x^2 - xy$ on R, and the points where these values occur.

DO NOT WRITE ABOVE THIS LINE!!

5. **(15 pt)** Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = x + y$ subject to the constraint $2x^2 + y^2 = 2$.

DO NOT WRITE ABOVE THIS LINE!!

6. **(15 pt)** Consider the integral

$$\int_0^2 \int_{2x}^4 2xy \, dy \, dx.$$

(a) Sketch the region of integration.

(b) Reverse the order of integration, by rewriting the bounds and converting it to an integral $dx\,dy$.

(c) Evaluate the integral (you can use any order of integration).

DO NOT WRITE ABOVE THIS LINE!!

7. **(10 pt)** Let $D = \{(x, y) : x^2 + y^2 \leq 4 \text{ and } y \geq -x\}$.

   (a) Sketch the region D.

   (b) Express D in polar coordinates via $D = \{(r, \theta) : a \leq r \leq b;\ \alpha \leq \theta \leq \beta\}$.

   (c) Evaluate $\iint_D e^{x^2+y^2}\, dA$.

DO NOT WRITE ABOVE THIS LINE!!

8. **(10 pt)** A tetrahedron is bounded by the xy-plane, yz-plane, xz-plane and the plane

$$x + 2y + 4z = 8.$$

Write down an iterated triple integral that expresses the volume of the tetrahedron. Do not evaluate the integral.