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MATH 210 Exam 2

March 22, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Lukina			Braithwaite	
Cameron			Kobotis	
Abramov			Shulman	
Heard			Woolf	
Skalit			Freitag	

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1. **(10pt)** Give an equation for the plane tangent to the surface defined by

$$xy + xz - yz = 1$$

at the point $P(2, -3, 1)$.

$F(x, y, z) = xy + xz - yz - 1 = 0$

$\nabla F = \langle y+z, x-z, x-y \rangle$

$\nabla F(2, -3, 1) = \langle -3+1, 2-1, 2-(-3) \rangle =$

$\langle -2, 1, 5 \rangle$

$-2(x-2) + (y+3) + 5(z-1) = 0$

2. **(15 pt)** Find all critical points of the function

$$f(x, y) = 2xy - 2x^2 - y^3 + 3$$

and classify them using the Second Derivative Test.

$f_x = 2y - 4x$ $\qquad$ $f_y = 2x - 3y^2$

$2y - 4x = 0$ $\qquad$ $y = 2x$

$2x - 3y^2 = 0$

$2x - 3 \cdot 4x^2 = 0$

$x - 6x^2 = 0$

$x(1 - 6x) = 0$

$x = 0$ or $1 - 6x = 0$

$x = \frac{1}{6}$

if $x = 0$, $y = 0$, $(0,0)$ critical point

if $x = \frac{1}{6}$, $y = \frac{2}{6} = \frac{1}{3}$, $(\frac{1}{6}, \frac{1}{3})$ critical point

$f_{xx} = -4$, $f_{xy} = 2$, $f_{yy} = -6y$

$D(x, y) = 24y - 4$

$D(0,0) = -4 < 0$ saddle point

$D(\frac{1}{6}, \frac{1}{3}) = \frac{24}{3} - 4 = 8 - 4 > 0$

$f_{xx}(\frac{1}{6}, \frac{1}{3}) \neq -4 < 0$, local max

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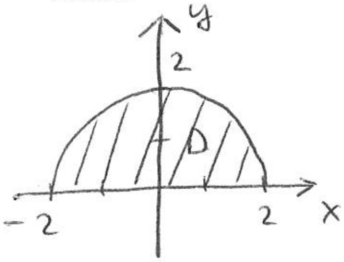
3. **(15 pt)** Let D be the half-disk $D = \{(x, y) \mid -2 \le x \le 2, 0 \le y \le \sqrt{4 - x^2}\}$, and let

$$f(x, y) = xy^2 - x.$$

Find the absolute minimum and maximum values achieved by f on D, and the points where they occur.



critical points:

$f_x = y^2 - 1 = 0 \qquad y = \pm 1,$

$f_y = 2xy = 0 \qquad x = 0,$

points $(0,-1)$, $(0,1)$,

$(0,-1)$ is not in D

$y = \sqrt{4-x^2}, \quad -2 \le x \le 2$

$g(x) = x(4-x^2) - x = 4x - x^3 - x = 3x - x^3$

$g'(x) = 3 - 3x^2 = 0, \quad x^2 = 1, \quad x = \pm 1,$

points $(1, \sqrt{3})$, $(-1, \sqrt{3})$

$y = 0: \quad -2 \le x \le 2, \quad g(x) = -x, \quad g'(x) = -1 \ne 0$

no critical points.

$f(0,1) = 0$

$f(1, \sqrt{3}) = 1 \cdot 3 - 1 = 2$

$f(-1, \sqrt{3}) = -1 \cdot 3 + 1 = -3 + 1 = -2$

$f(-2, 0) = 2 \qquad f(2, 0) = -2$

abs max $\quad f(1, \sqrt{3}) = f(-2, 0) = 2$

abs min $\quad f(2, 0) = f(-1, \sqrt{3}) = -2$

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4. **(10 pt)** Use the method of Lagrange multipliers to find the maximum and the minimum values of the function

$$f(x, y) = x^2 - y^2$$

on the ellipse $4x^2 + y^2 = 1$.

$\nabla f = \langle 2x, -2y \rangle$ $\nabla g = \langle 8x, 2y \rangle$

$2x = \lambda 8x$

$-2y = \lambda 2y$ $\lambda y + y = y(\lambda + 1) = 0$

$4x^2 + y^2 = 1$ $y = 0$ $\lambda = -1$

$y = 0$: $4x^2 = 1$, $x^2 = \frac{1}{4}$, $x = \pm \frac{1}{2}$,

$(-\frac{1}{2}, 0)$, $(\frac{1}{2}, 0)$

$\lambda = -1$: $x = -4x$, $x = 0$, $y^2 = 1$, $y = \pm 1$,

$(0, -1)$, $(0, 1)$

$f(0, -1) = -1$, $f(0, 1) = -1$

$f(-\frac{1}{2}, 0) = \frac{1}{4}$ $f(\frac{1}{2}, 0) = \frac{1}{4}$

min $f(0, \pm 1) = -1$,

max $f(\pm \frac{1}{2}, 0) = \frac{1}{4}$

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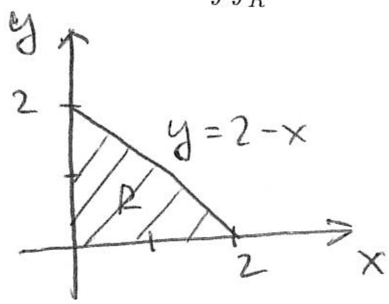
5. **(15 pt)** Let R be the region in the first quadrant of the xy-plane bounded by the coordinate axes and the line $y = 2 - x$.

   (a) Sketch R.

   (b) Evaluate $\iint_R 3x\, dA$.



$R = \{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le 2-x\}$

$$\int_0^2 \int_0^{2-x} 3x\, dy\, dx =$$

$$\int_0^2 3xy \Big|_0^{2-x} dx =$$

$$\int_0^2 3x(2-x-0)\,dx = \int_0^2 (6x - 3x^2)\,dx =$$

$$\left(6\frac{x^2}{2} - 3\frac{x^3}{3}\right)\Big|_0^2 = (3x^2 - x^3)\Big|_0^2 =$$

$$3 \cdot 4 - 8 = 4$$

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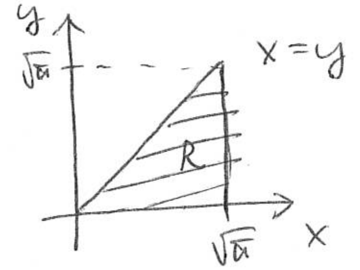
6. **(10 pt)** Consider the integral

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2)\, dx\, dy.$$

(a) Sketch the region of integration and change the order of integration to $dy\, dx$.

(b) Evaluate the integral (use the order of integration $dy\, dx$).

a) 

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin x^2\, dx\, dy =$$

$$\int_0^{\sqrt{\pi}} \int_0^{x} \sin x^2\, dy\, dx$$

b) $$\int_0^{\sqrt{\pi}} \int_0^{x} \sin x^2\, dy\, dx = \int_0^{\sqrt{\pi}} y \sin x^2 \Big|_0^x\, dx =$$

$$\int_0^{\sqrt{\pi}} x \sin x^2\, dx$$

$u = x^2 \qquad du = 2x\,dx$

if $x = 0$ then $u = 0$

$x = \sqrt{\pi} \qquad u = \pi$

$$\int_0^{\sqrt{\pi}} x \sin x^2\, dx = \frac{1}{2} \int_0^{\pi} \sin u\, du = \frac{1}{2} (-\cos u)\Big|_0^{\pi} =$$

$$\frac{1}{2}(-\cos \pi + \cos 0) = 1$$

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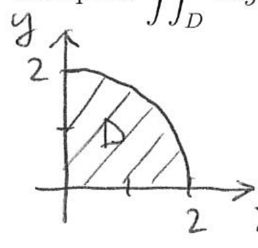
7. **(15 pt)** Consider the quarter-disk $D = \{(x, y) \mid 0 \leq x \leq 2,\ 0 \leq y \leq \sqrt{4 - x^2}\}$.

(a) Sketch D as a shaded region in the xy-plane.

(b) Express D in polar coordinates.

(c) Compute $\iint_D 4xy\, dA$.

a)



b) $D = \{(r, \theta) \mid 0 \leq \theta \leq \frac{\pi}{2},\ 0 \leq r \leq 2\}$

c) $\iint\limits_D 4xy\, dA = \int_0^{\frac{\pi}{2}} \int_0^2 4r^2 \cos\theta \sin\theta\, r\, dr\, d\theta =$

$\int_0^{\frac{\pi}{2}} \int_0^2 4r^3 \cos\theta \sin\theta\, dr\, d\theta =$

$\int_0^{\frac{\pi}{2}} \cos\theta \sin\theta\, r^4 \Big|_0^2\, d\theta =$

$\int_0^{\frac{\pi}{2}} 16 \cos\theta \sin\theta\, d\theta = 8 \int_0^{\frac{\pi}{2}} \sin 2\theta\, d\theta =$

$8\, \frac{1}{2} (-\cos 2\theta) \Big|_0^{\frac{\pi}{2}} =$

$4(-\cos\pi + \cos 0) = 8$

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8. (~~**10**~~ 5 **pt**) Rewrite the following integral in cylindrical coordinates. Do not evaluate the integral.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} 2z\,dz\,dy\,dx.$$

$$\int_{0}^{2\pi}\int_{0}^{1}\int_{0}^{\sqrt{1-r^2}} 2zr\,dz\,dr\,d\theta .$$