MATH 210
Sample exam problems for the 2nd hour exam
Fall 2009

1. Let $f(x,y) = 3x^2 + xy + 2y^2$. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ at $(1,1)$, and find the best linear approximation of f at $(1,1)$ and use it to estimate $f(1.1, 1.2)$.

2. Find and classify the critical points of the function $f(x,y) = x^3 - 3xy + y^3$.

3. Sketch the region of integration for the integral $\int_0^4 \int_{\sqrt{y}}^2 \sin(x^3)\, dx\, dy$. Compute the integral.

4. Find the minimum and maximum of the function $f(x,y,z) = x + y - z$ on the ellipsoid
$$R = \left\{ (x,y,z) : \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \right\}.$$

5. Find the tangent plane to the surface:
$$S = \{(x,y,z) : x^2 + y^3 - 2z = 1\}$$
at the point $(1,2,4)$.

6. Let $F(x,y,z) = 3x^2 + y^2 - 4z^2$. Find the equation of the tangent plane to the level surface $F(x,y,z) = 1$ at the point $(1,-4,3)$.

7. Let $f(x,y) = \frac{1}{3}x^3 + y^2 - xy$. Find all critical points of $f(x,y)$ and classify each as a local maximum, local minimum, or saddle point.

8. Find the maximum and minimum of the function $f(x,y) = x^2 - y$ subject to the condition $x^2 + y^2 = 4$.

9. Use polar coordinates to find the volume of the region bounded by the paraboloid $z = 1 - x^2 - y^2$ in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$.

10. Find the minimum and the maximum of the function
$$f(x,y,z) = x^2 - y^2 + 2z^2$$
on the surface of the sphere defined by the equation $x^2 + y^2 + z^2 = 1$.

11. Using cylindrical coordinates, compute

$$\iiint_W (x^2+y^2)^{\frac{1}{2}}\, dV$$

where W is the region within the cylinder $x^2+y^2 \le 4$ and $0 \le z \le y$.

12. Compute the integral $\iiint_B x^2\, dV$, where B is the unit ball

$$B = \{(x,y,z) :\ x^2+y^2+z^2 \le 1\}.$$

13. Find the volume of the region bounded below and above by the surfaces $z = x^2+y^2$ and $z = 2-x^2-y^2$.

14. Let $f(x,y) = e^{xy}$ and (r,θ) be polar coordinates. Find $\dfrac{\partial f}{\partial r}$. Express your answer in terms of the variables x and y.

15. Compute the average value of the function $f(x,y) = 2+x-y$ on the quarter disk $A = \{(x,y) :\ x \ge 0,\ y \ge 0,\ x^2+y^2 \le 1\}$.

16. Compute the integral

$$\iint_D \frac{x}{y+1}\, dA$$

where D is the triangle with vertices $(0,0), (1,1), (2,0)$.

17. Let $f(x,y) = x^2-x+y^2$, and let $\mathcal{D}$ be the bounded region defined by the inequalities $x \ge 0$ and $x \le 1-y^2$.

    (a) Find and classify the critical points of $f(x,y)$.

    (b) Sketch the region $\mathcal{D}$.

    (c) Find the absolute maximum and minimum values of f on the region $\mathcal{D}$, and list the points where these values occur.

18. Consider the function $F(x,y) = x^2 e^{4x-y^2}$. Find the direction (unit vector) in which F has the fastest growth at the point $(1,2)$.

19. Let $\vec{\mathbf{r}}(t) = \langle e^{-t}, \cos(t)\rangle$ describe movement of a point in the plane, and let $f(x,y) = x^2y - e^{x+y}$. Use the chain rule to compute the derivative of $f(\vec{\mathbf{r}}(t))$ at time $t=0$.

20. Let the function $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ describe the density in the region $A = \{x^2+y^2+z^2 \le 1,\ \sqrt{x^2+y^2} \le z\}$. Use spherical coordinates to compute its mass.