1. Let $f(x, y) = 3x^2 + xy + 2y^2$. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ at $(1, 1)$, and find the best linear approximation of $f$ at $(1, 1)$ and use it to estimate $f(1.1, 1.2)$.

2. Find and classify the critical points of the function $f(x, y) = x^3 - 3xy + y^3$.

3. Sketch the region of integration for the integral $\int_0^4 \int_{\sqrt[4]{x}}^2 \sin(x^3) \, dx \, dy$. Compute the integral.

4. Find the minimum and maximum of the function $f(x, y, z) = x + y - z$ on the ellipsoid

$$R = \left\{ (x, y, z) : \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \right\}.$$

5. Find the tangent plane to the surface:

$$S = \{(x, y, z) : x^2 + y^3 - 2z = 1\}$$
at the point $(1, 2, 4)$.

6. Let $F(x, y, z) = 3x^2 + y^2 - 4z^2$. Find the equation of the tangent plane to the level surface $F(x, y, z) = 1$ at the point $(1, -4, 3)$.

7. Let $f(x, y) = \frac{1}{3}x^3 + y^2 - xy$. Find all critical points of $f(x, y)$ and classify each as a local maximum, local minimum, or saddle point.

8. Find the maximum and minimum of the function $f(x, y) = x^2 - y$ subject to the condition $x^2 + y^2 = 4$.

9. Use polar coordinates to find the volume of the region bounded by the paraboloid $z = 1 - x^2 - y^2$ in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$.

10. Find the minimum and the maximum of the function

$$f(x, y, z) = x^2 - y^2 + 2z^2$$
on the surface of the sphere defined by the equation $x^2 + y^2 + z^2 = 1$. 


11. Using cylindrical coordinates, compute
\[
\iiint_W (x^2 + y^2)^{\frac{1}{2}} \, dV
\]
where \( W \) is the region within the cylinder \( x^2 + y^2 \leq 4 \) and \( 0 \leq z \leq y \).

12. Compute the integral \( \iiint_B x^2 \, dV \), where \( B \) is the unit ball
\[
B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}.
\]

13. Find the volume of the region bounded below and above by the surfaces
\[
z = x^2 + y^2 \quad \text{and} \quad z = 2 - x^2 - y^2.
\]

14. Let \( f(x, y) = e^{xy} \) and let \( (r, \theta) \) be polar coordinates. Find \( \frac{\partial f}{\partial r} \). Express your answer in terms of the variables \( x \) and \( y \).

15. Compute the average value of the function \( f(x, y) = 2 + x - y \) on the quarter disk \( A = \{(x, y) : x \geq 0, \ y \geq 0, \ x^2 + y^2 \leq 1\} \).

16. Compute the integral
\[
\int_D \frac{x}{y + 1} \, dA
\]
where \( D \) is the triangle with vertices \((0, 0), (1, 1), (2, 0)\).

17. Let \( f(x, y) = x^2 - x + y^2 \), and let \( D \) be the bounded region defined by the inequalities \( x \geq 0 \) and \( x \leq 1 - y^2 \).
   
   (a) Find and classify the critical points of \( f(x, y) \).
   
   (b) Sketch the region \( D \).
   
   (c) Find the absolute maximum and minimum values of \( f \) on the region \( D \), and list the points where these values occur.

18. Consider the function \( F(x, y) = x^2 e^{4x-y^2} \). Find the direction (unit vector) in which \( F \) has the fastest growth at the point \((1, 2)\).

19. Let \( \vec{r}(t) = (e^{-t}, \cos(t)) \) describe movement of a point in the plane, and let \( f(x, y) = x^2 y - e^{x+y} \). Use the chain rule to compute the derivative of \( f(\vec{r}(t)) \) at time \( t = 0 \).

20. Let the function \( f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \) describe the density in the region \( A = \{x^2 + y^2 + z^2 \leq 1, \ \sqrt{x^2 + y^2} \leq z\} \). Use spherical coordinates to compute its mass.