MATH 220 Final Exam Fall 2011

1	2	3	4	5	6	7	8	9	10	11	12
(**20**)	(**20**)	(**20**)	(**15**)	(**15**)	(**15**)	(**15**)	(**15**)	(**15**)	(**15**)	(**15**)	(**20**)

SCORE _____ / 200

Name (print) _____ **Signature** _____

Instructor (circle one) **Greenblatt** **Knessl** **Nicholls**

Some potentially useful formulas:

$$\int x\sin(Ax)dx = -\frac{x}{A}\cos(Ax)+\frac{1}{A^2}\sin(Ax), \qquad \int x\cos(Ax)dx = \frac{x}{A}\sin(Ax)+\frac{1}{A^2}\cos(Ax)$$

$$y_2(t) = y_1(t)\int^t [y_1(u)]^{-2}\exp\left(-\int^u p(v)dv\right)du$$

$$y_p(t) = y_1(t)\int^t \frac{-g(u)y_2(u)}{W[y_1,y_2](u)}du+y_2(t)\int^t \frac{g(u)y_1(u)}{W[y_1,y_2](u)}du, \quad W[y_1,y_2](t) = y_1(y)y_2'(t)-y_1'(t)y_2(t)$$

$$\mathcal{L}\{1\} = \frac{1}{s}, \quad \mathcal{L}\left\{e^{at}\right\} = \frac{1}{s-a}, \quad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad \mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2}, \quad \mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2},$$

$$\mathcal{L}\left\{e^{at}t^n\right\} = \frac{n!}{(s-a)^{n+1}}, \quad \mathcal{L}\left\{e^{at}f(t)\right\} = \mathcal{L}\{f\}(s-a) = F(s-a),$$

$$\mathcal{L}\{f'(t)\} = sF(s)-f(0), \quad \mathcal{L}\{f''(t)\} = s^2F(s)-sf(0)-f'(0),$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s), \quad \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}, \quad \mathcal{L}\{\delta(t-a)\} = e^{-as}$$

1. (8+12) Consider the equation

$$y'' + y = t^2 + e^t.$$

(a) Find the general solution to the homogeneous equation (with right hand side replaced by 0).

(b) Find a particular solution to the inhomogeneous equation.

2. (10+10) Solve

(a) $\dfrac{dy}{dx} = \dfrac{y}{x} + xe^x,$ (b) $\dfrac{dy}{dx} = \dfrac{\sin x - ye^{xy}}{xe^{xy} + 1}.$

3. (20) Use the Euler method with *two* steps to approximate $y(1)$, where

$$y' = (1 + x)y + x^2, \qquad y(0) = 1.$$

4. (3+12) Let $p(t)$ be the population of some species, which satisfies the ODE

$$p' = 3p - p^2.$$

(a) If $p(0) = 0$ what is $p(t)$ for $t > 0$?

(b) Solve for $p(t)$ for the initial condition $p(0) = 1$, and then evaluate the limit of $p(t)$ as $t \to \infty$.

5. (8+7) Solve

(a) $t^2y'' - ty' + y = 0,$ (b) $t^2y'' + ty' + 4y = 0.$

6. (3+12) Consider the ODE

$$y'' + 2ty' - 4y = 0.$$

(a) Is $y_1(t) = 2t^2 + 1$ a solution ?

(b) Find the general solution. (Hint: Consider the method of reduction of order. You may leave your answer as an integral.)

7. (15) Find the general solution to

$$y'' - 2y' + y = \frac{2e^t}{t^3}.$$

8. (15) Find the general solution to the system

$$x' = 2x + y, \qquad y' = x + 2y.$$

9. (15) Use Laplace transforms to solve

$$y'' + y' = e^t, \qquad y(0) = 1, \quad y'(0) = 0.$$

To receive any credit, you **must** use Laplace transforms.

10. (15) Use Laplace transforms to solve

$$y'' + 4y = \delta(t-1), \qquad y(0) = 0, \quad y'(0) = 1.$$

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11. (15) Solve the heat equation

$$u_t = \beta u_{xx}\ ; \quad 0 < x < 1, \ \ t > 0$$

$$u(0,t) = 0, \qquad u(1,t) = 0$$

$$u(x,0) = 2\sin(\pi x) + \sin(4\pi x).$$

Start by writing the general solution that satisfies the PDE and boundary conditions, and then find the solution that also satisfies this initial condition.

12. (3+2+3+12) Let $f(x) = 2x$ for $0 < x < 1$.

(a) Sketch the *even* extension of $f(x)$ over the range $-1 < x < 1$.

(b) Does the Fourier series of (the extended) $f(x)$ involve sines, cosines or both ?

(c) What is the average value of (the extended) $f(x)$ over a period ?

(d) Compute explicitly the Fourier series of (the extended) $f(x)$.