Math 220 Final – December 10, 2015

Name: ______________________________ **UIN:** ____________________

Instructor Name: ______________________ **Lecture Time:** ____________

Instructions:

- During the exam, you may **not** use your books, notes, reference materials, or **any electronic devices**, including calculators and cell phones. Violating this rule will result in expulsion from the exam and a score of zero (0)!
- **No form of reproduction or provision of this exam**, or any part thereof, including, but not limited to, copying for personal use, sharing with current or prospective students, or posting on the Internet in open access or restricted selective spaces **is permitted without the written permission of the course coordinator**, Prof. Alexey Cheskidov, Math 220, Fall 2015.
- You are required to show your work on each problem on this exam. **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanations, and/or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

Scores:

1. __________/**15 points**

2. __________/**10 points**

3. __________/**10 points**

4. __________/**15 points**

5. __________/**15 points**

6. __________/**10 points**

7. __________/**15 points**

8. __________/**15 points**

9. __________/**15 points**

Total: __________/**120 points**

$$\mathcal{L}\big\{e^{at}t^n\big\}(s) = \frac{n!}{(s-a)^{n+1}}\,, \qquad \mathcal{L}\big\{e^{at}\sin(bt)\big\}(s) = \frac{b}{(s-a)^2+b^2}\,, \qquad \mathcal{L}\big\{e^{at}\cos(bt)\big\}(s) = \frac{s-a}{(s-a)^2+b^2}\,,$$

$$\mathcal{L}\big\{f'(t)\big\}(s) = s\mathcal{L}\big\{f(t)\big\}(s) - f(0)\,, \qquad \mathcal{L}\big\{f(t-a)u(t-a)\big\}(s) = e^{-as}\mathcal{L}\big\{f(t)\big\}(s)\,, \qquad \mathcal{L}\big\{\delta(t-a)\big\}(s) = e^{-as}\,.$$

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1. **(15 points).** Solve the initial value problem

$$y'' + 2y' + y = \sin(t) - \delta(t-1)\,, \qquad y(0) = 0\,, \quad y'(0) = 1\,.$$

2. **(10 points).** Compute the Laplace transform of the function

$$f(t) = te^{-2t} \sin\left(t + \frac{\pi}{2}\right).$$

3. **(10 points)** Solve the following initial value problem:

$$y' = e^{x^3} - \frac{2y}{x}, \qquad y(1) = \frac{2e}{3}.$$

4. **(15 points).** Find the (implicit) solution to

$$\left(\frac{y}{x} + x^3\right)\,dx + \left(y^2 + \ln x\right)dy = 0, \qquad y(1) = 1.$$

5. **(15 points).** A nitric acid solution flows at a constant rate of 6 L/min into a large tank that initially held 200 L of a 0.5% nitric acid solution. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 8 L/min. If the solution entering the tank is 20% nitric acid, determine the volume of nitric acid in the tank after t min.

6. **(10 points).** Consider the following initial value problem:

$$\frac{dy}{dx} = x^2 - xy, \qquad y(0) = 1.$$

Use the Euler method with two steps to approximate $y(1)$.

7. **(15 points).** Find a general solution to

$$y'' - y = t + e^t + \sin t.$$

8. **(15 points).** Consider the wave equation

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}(x,t) = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < L\,, \quad t > 0\,, \\ \\ u(0,t) = u(L,t) = 0 & t > 0\,. \end{cases}$$

(a) Derive a formula for the general solution $u(x,t)$ of this wave equation. (*Hint:* use separation of variables or Fourier series.)

(b) Find the solution $u(x,t)$ satisfying the initial values:

$$u(x,0) = \sin\left(\frac{\pi x}{L}\right), \qquad \frac{\partial u}{\partial t}(x,0) = \sin\left(\frac{2\pi x}{L}\right), \qquad 0 < x < L\,.$$

9. **(15 points).** Consider the function

$$f(x) = 1 - x, \qquad 0 < x < 1.$$

(a) Sketch the odd extension of $f(x)$ over the range $-1 < x < 1$.

(b) Does the Fourier series of the extended $f(x)$ involve only sines, only cosines, or both sines and cosines?

(c) Compute explicitly the Fourier series of the extended $f(x)$.

EXTRA SPACE TO WORK – IF YOU USE THIS PAGE TO SOLVE SOME OF THE PROBLEMS, PLEASE **MARK THIS CLEARLY** BOTH HERE AND IN THE SPACE ASSIGNED TO THE PROBLEM YOU ARE SOLVING!