

MATHEMATICS 220: EXAM I
University of Illinois at Chicago (Professor Nicholls)
October 3, 2014

Please read the exam carefully and follow all instructions. **SHOW ALL OF YOUR WORK.** Please put a box around your final answer.

1. (20 points) Solve the initial value problem

$$\frac{dy}{dx} = -\frac{x^2}{2y}, \quad y(0) = 2.$$

2. (20 points) Obtain the general solution to

$$(t + y + 1)dt - dy = 0.$$

3. (20 points) Find the general solution of the differential equation

$$z''(t) + z'(t) - z(t) = 0.$$

4. (20 points) A swimming pool whose volume is 10,000 gal contains water that is 0.1 % chlorine. Starting at $t = 0$, the city pumps pure water into the pool at a rate of 10 gal/min. The pool water flows out at the same rate.

- (a) (4 points) How much chlorine is in the tank at $t = 0$ (before the city water enters)?
- (b) (4 points) Write down an Initial Value Problem for the amount $A = A(t)$ of chlorine at time $t > 0$.
- (c) (12 points) Solve this Initial Value Problem for $A(t)$.

5. (20 points) Find the general solution of the differential equation

$$\frac{dy}{dx} = -\frac{y^2 \cos(x)e^{\sin(x)}}{2ye^{\sin(x)} - \pi \sin(\pi y)}.$$