MATHEMATICS 220: EXAM II
University of Illinois at Chicago (Professor Nicholls)
April 3, 2015

Please read the exam carefully and follow all instructions. **SHOW ALL OF YOUR WORK.** Please put a box around your final answer.

1. (20 points) Using the **Method of Variation of Parameters** (**any other method will receive zero points**) find the general solution to the differential equation
$$\frac{d^2y}{d\theta^2} + 16y = \sec(4\theta).$$

2. (20 points) Find the solution to the initial value problem
$$t^2y''(t) - 5ty'(t) + 9y(t) = 0, \quad y(1) = 2, \quad y'(1) = 2.$$

3. (20 points) Solve the phase plane equation for the given system
$$\frac{dx}{dt} = x^2 - 2y^{-3}$$
$$\frac{dy}{dt} = 3x^2 - 2xy.$$

4. (20 points) Find the inverse Laplace transform of
$$F(s) = \frac{3s - 3\pi - 4}{s^2 - 2\pi s + \pi^2}.$$

5. (20 points) Solve the given initial value problem using the **Method of Laplace Transforms** (**any other method will receive zero points**):
$$y''(t) + 6y'(t) + 5y(t) = 16e^{-5t}, \quad y(0) = 0, \quad y'(0) = 0.$$

List of Laplace Transforms

1. $\mathcal{L}\{1\} = \dfrac{1}{s}, \quad s > 0$
2. $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}, \quad s > a$
3. $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}, \quad s > 0$
4. $\mathcal{L}\{\sin(bt)\} = \dfrac{b}{s^2+b^2}, \quad s > 0$
5. $\mathcal{L}\{\cos(bt)\} = \dfrac{s}{s^2+b^2}, \quad s > 0$
6. $\mathcal{L}\{e^{at}t^n\} = \dfrac{n!}{(s-a)^{n+1}}, \quad s > a$
7. $\mathcal{L}\{e^{at}\sin(bt)\} = \dfrac{b}{(s-a)^2+b^2}, \quad s > a$
8. $\mathcal{L}\{e^{at}\cos(bt)\} = \dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
9. $\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s-a)$
10. $\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
11. $\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
12. $\mathcal{L}\left\{f^{(n)}\right\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
13. $\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \dfrac{d^n}{ds^n}\mathcal{L}\{f\}(s)$
14. $\mathcal{L}\{f(t-a)u(t-a)\}(s) = e^{-as}F(s)$
15. $\mathcal{L}\{u(t-a)\}(s) = \dfrac{e^{-as}}{s}$
16. $\mathcal{L}\{g(t)u(t-a)\}(s) = e^{-as}\mathcal{L}\{g(t+a)\}(s)$
17. If f has period T then

$$\mathcal{L}\{f\}(s) = \frac{F_T(s)}{1-e^{-sT}} = \frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$$

18. $\mathcal{L}\{\delta(t-a)\}(s) = e^{-as}$