Math 220 Midterm 2 – November 6, 2015

Name: _____ **UIN:** _____

Problem session day: _____ **Problem session time:** _____

Instructions:

- During the exam, you may **not** use your books, notes, reference materials, or **any electronic devices**, including calculators and cell phones. Violating this rule will result in expulsion from the exam and a score of zero (0)!
- **No form of reproduction or provision of this exam**, or any part thereof, including, but not limited to, copying for personal use, sharing with current or prospective students, or posting on the Internet in open access or restricted selective spaces **is permitted without the written permission of the course coordinator**, Prof. Alexey Cheskidov, Math 220, Fall 2015.
- You are required to show your work on each problem on this exam. **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanations, and/or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Write your solutions in the exam booklets provided. At the end of the exam, you **must return this problem sheet together with your exam booklet.**

Scores:

1. _____/3 points
2. _____/4 points
3. _____/5 points
4. _____/3 points
5. _____/5 points

Total: _____/20 points

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1. **(3 points)** Find the general solution of the following differential equation:

$$4t^2y'' + 8ty' + y = 0, \quad \text{for } t > 0.$$

2. **(4 points)** Find the general solution of the differential equation:

$$y'' + 4y = 8\cos(2t)\,.$$

3. **(5 points)** Find the solution to the following system of ordinary differential equations

$$\begin{cases} x' = 4x + y + e^t, \\ y' = -2x + y. \end{cases}$$

4. **(3 points)** Compute the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$ of the function

$$f(t) = e^{7t}\sin^2(t)\,,$$

and indicate for which $s \in \mathbb{R}$ the $\mathcal{L}$–transform exists.

5. **(5 points)**

(a) Compute the inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}(t)$ of the function $F(s)$ such that:

$$s^2F(s) - 4F(s) = \frac{5}{s^2+1}\,.$$

(b) Find the solution to the initial value problem

$$y'' - 4y = 5\sin(t)\,, \quad y(0) = 0\,, \;\; y'(0) = 0\,.$$