Math 310 Midterm 2 - 17 March 2010

NO WORK = NO CREDIT. NO CALCULATOR.

Please print your name and UIN, and sign the following academic honesty disclosure:

I affirm that I have never given nor received aid on this examination. I understand that cheating is a violation of the student code. Cheating will be a reason for a grade of F in the course and referral to proper officials for possible further disciplinary action.

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1. (20 pts) Consider the set $S = \mathbb{R}$ consisting of all real numbers. On the set $S$, consider the operations $\oplus, \otimes$ defined as:

\[ u \oplus v = \max(u, v) \]
\[ k \otimes u = ku \]

(a) Determine whether vector addition is commutative; i.e. $u \oplus v = v \oplus u$ for all $u, v \in S$.
(b) Determine whether or not $S$ has a zero element $\mathbf{0}$ such that $u \oplus \mathbf{0} = u$ for all $u \in S$.

2. (50 pts) Consider the matrix $A$, with its reduced row echelon form $U$, given below.

\[
A = \begin{pmatrix}
2 & -4 & 3 & -1 & 3 & 1 \\
3 & -6 & 3 & 3 & 0 & -3 \\
-5 & 10 & -3 & -11 & 6 & 11 \\
3 & -6 & 3 & 3 & -3 & 3
\end{pmatrix}
\quad \xrightarrow{\text{RREF}} U = \begin{pmatrix}
1 & -2 & 0 & 4 & 0 & -10 \\
0 & 0 & 1 & -3 & 0 & 9 \\
0 & 0 & 0 & 0 & 1 & -2 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(a) Find a basis for the rowspace of $A$.
(b) Find a basis for the nullspace of $A$.
(c) Are the columns of $A$ linearly independent? If not, indicate any dependency relations amongst them.
(d) Do the columns of $A$ span $\mathbb{R}^4$? Explain.
(e) What is the dimension of $R(A)$? Explain

3. (10 pts) Find $S^\perp$, the orthogonal complement of $S$:

\[ S = \text{Span} \left( \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \\ 2 \\ 4 \end{pmatrix} \right) \]

4. (20 pts) Determine whether each of the following sets are subspaces of $\mathbb{R}^{2 \times 2}$:

(a) The set $S_1$ of all triangular $2 \times 2$ matrices.
(b) The set $S_2$ of all symmetric $2 \times 2$ matrices.