Math 310 Midterm 2 - 17 March 2010

NO WORK = NO CREDIT. NO CALCULATOR.

Please print your name and UIN, and sign the following academic honesty disclosure:

I affirm that I have never given nor received aid on this examination. I understand that cheating is a violation of the student code. Cheating will be a reason for a grade of F in the course and referral to proper officials for possible further disciplinary action.

Name:									
UIN:									
Signature:									

#	Score
1	/20
2	/50
3	/10
4	/20
T	/100

1. (20 pts) Consider the set $S = \mathbb{R}$ consisting of all real numbers. On the set S, consider the operations $\oplus, \otimes$ defined as:

$$\mathbf{u} \oplus \mathbf{v} = max(u, v)$$
$$k \otimes \mathbf{u} = ku$$

   (a) Determine whether vector addition is commutative; *i.e.* $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$ for all $\mathbf{u}, \mathbf{v} \in S$.

   (b) Determine whether or not S has a zero element $\mathbf{0}$ such that $\mathbf{u} \oplus \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in S$.

2. (50 pts) Consider the matrix A, with its reduced row echelon form U, given below.

$$A = \begin{pmatrix} 2 & -4 & 3 & -1 & 3 & 1 \\ 3 & -6 & 3 & 3 & 0 & -3 \\ -5 & 10 & -3 & -11 & 6 & 11 \\ 3 & -6 & 3 & 3 & -3 & 3 \end{pmatrix} \xrightarrow{\text{RREF}} U = \begin{pmatrix} 1 & -2 & 0 & 4 & 0 & -10 \\ 0 & 0 & 1 & -3 & 0 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   (a) Find a basis for the rowspace of A.

   (b) Find a basis for the nullspace of A.

   (c) Are the columns of A linearly independent? If not, indicate any dependency relations amongst them.

   (d) Do the columns of A span $\mathbb{R}^4$? Explain.

   (e) What is the dimension of $R(A)$? Explain

3. (10 pts) Find $S^\perp$, the orthogonal complement of S:

$$S = \text{Span}\left(\begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \\ 2 \\ 4 \end{pmatrix} \right)$$

4. (20 pts) Determine whether each of the following sets are subspaces of $\mathbb{R}^{2\times 2}$:

   (a) The set S_1 of all triangular 2×2 matrices.

   (b) The set S_2 of all symmetric 2×2 matrices.