Math310:Exam 2
Spring 2009

Problem 1. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 2 & -2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$.

(15pts) a) Find a basis of the row space of A. Find a basis of the column space of A. Determine the rank of A.

(10pts) b) Find a basis of $N(A)$. What is the dimension of $N(A)$?

(10pts) c) Let $L : \mathbb{R}^3 \to R^4$ be a map defined as follows: if $x \in \mathbb{R}^3$ then $L(x) = Ax$. Show that L is a linear map.

Problem 2. (15pts) Consider the following vectors in $M^{2\times 2}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}.$$

Are they linearly independent?

Problem 3. Let $F = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \right\}$ and $G = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ be two bases of $\mathbb{R}^2$.

(10pts) a) Find the transition matrix corresponding to the change of basis from F to G.

(10pts) b) If $w = 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ find the coordinate representation $[w]_G$ of w with respect to G.

Problem 4. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map such that

$$L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} -2x + y \\ 2x - y \end{pmatrix}.$$

(10pts) a) Determine the kernel and range of L.

(10pts) b) Find the standard matrix representation of L.

(15pts) c) Find the matrix representing L with respect to the fixed basis F, if $F = \left\{ \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \end{pmatrix} \right\}$.